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BY

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THESIS

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DIRECT-SEQUENCE SPREAD-SPECTRUM SIGNALING WITH APPLICATIONS TO PACKET RADIO SYSTEMS

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The application of direct-sequence spread-spectrum signaling to packet radio systems is considered. In particular, the performance of a receiver of information conveyed through a specular multipath channel is examined. It is shown that direct-sequence spread-spectrum signaling is a useful technique for resolving the several received signal replicas from the specular multipath channel and for combining the information which is inherent in the several received signal replicas. This signaling technique is also shown to be effective in combating the adverse effects of intersymbol interference and multiple-access interference.

Methods of evaluating the average probability of bit error of the multipath-combining receiver are investigated. Two different approximations to the average probability of bit error are developed. One approximation requires very little computation. The other approximation is appropriate for a wider range of system parameters although it requires more computation. The approximations are developed in a manner which identifies key parameters of the signature sequences. These key parameters which influence the performance can be used as a guide in selecting signature sequences. Attention is also given to systems which employ randomly generated signature sequences.

The performance of a receiver of information conveyed through the multiple-access channel is also investigated for the case of direct-sequence spread-spectrum signaling and randomly generated signature sequences. Methods of evaluating this performance in a computationally efficient manner are considered. One result is the development of upper and lower bounds on the average probability of bit error of a correlation receiver. Nice features of these bounds are that they require very low amounts of computation and that the upper and lower bounds can both be made arbitrarily tight by increasing the amount of computation that is performed. Furthermore, the types of computations which are required to evaluate the bounds are particularly suited to an array processor.

In the process of obtaining bounds on the average probability of bit error, bounds on the probability density function of the multiple-access interference are obtained. These are then used to study an approximation to the average probability of bit error. Finally, extensions of the approach for obtaining bounds to other signaling techniques and methods of demodulation are discussed.

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CHAPTER 1

INTRODUCTION

In a packet radio system a large number of asynchronous transmitters exist in a geographical region [1]. Each transmitter is silent most of the time, but each occasionally becomes active and transmits data to an intended receiver. When the data is sent, it is sent as a burst of digital data. After the burst is sent, the transmitter again becomes inactive or silent. The burst of data that is sent is called a packet. For example, a packet may consist of a group of a thousand consecutively transmitted data bits.

Many problems must be overcome for a packet radio system to work effectively. First of all, if two transmitters both try to transmit a packet of digital data simultaneously, a collision occurs. It can then happen that neither packet is received correctly. This is the problem of multiple-access interference. Another problem occurs because of the characteristics of the communication channel often encountered in practice. When a given transmitter sends a signal, several replicas of the signal appear at the input of a receiver. These replicas have varying associated delays, and some of the delays may be several times the data bit duration. Hence, signal replicas corresponding to different transmitted data bits may appear at the receiver simultaneously, and this causes severe problems of intersymbol interference.

Direct-sequence spread-spectrum signaling can help to solve several of the problems encountered in packet radio systems. Direct-sequence spread-spectrum signaling combats the intersymbol interference and provides multiple-access capability. Furthermore, this signaling technique can be used to resolve the multiple replicas of a transmitted signal that appear at the input to a receiver and to combine the information inherent in each of the signal replicas. This technique, called diversity combining, can greatly improve the performance of the receiver.

Previous analyses have dealt largely with direct-sequence spread-spectrum multiple-access (DS/SSMA) communications through an additive white Gaussian noise (AWGN) channel. The channel that is considered in these analyses is also sometimes called the multiple-access channel. In [2]-[9], various methods of ascertaining the performance of such systems are described. A review of several available approximations to the average probability of bit error is given in [5].

Fewer analyses of DS/SSMA communications through the specular multipath channel are available. In [10], a direct-sequence spread-spectrum system consisting of a single transmitter, a correlation receiver, and the specular multipath channel is analyzed in order to determine the signal-to-noise ratio at the receiver. In [11], the same system is analyzed by obtaining the characteristic function of the interference at the receiver. Both of these systems, however, involve a correlation receiver and a single transmitter. In [12], several receiver structures are suggested which are more complex than the correlation receiver, but which allow the information from the several received signal replicas from a transmitter to be combined and utilized. In order to resolve the signal replicas and combine the corresponding information, spread-spectrum signaling is suggested.

In this thesis, we consider the analysis of a multipath-combining receiver related to those suggested in [12]. The system we consider is a general system that models systems which use quadriphase-shift-keyed (QPSK), offset quadriphase-shift-keyed (OQPSK), and minimum-shift-keyed (MSK) modulation, as well as binary phase-shift-keyed (BPSK) modulation. We consider a multipath-combining receiver and determine the performance of the system not only for the case of a single transmitter, but also for the case of multiple interfering transmitters. Furthermore, we determine the performance of the system in terms of parameters of the signature sequences that are used in the system. These parameters can therefore be used as guides in selecting signature sequences for the system. Results are also given for the case of randomly generated signature sequences.

The numerical computations necessary to determine the exact performance of the multipath-combining receiver can be quite extensive if we assume a general model that accounts for all the random parameters of the system. We therefore consider a specialization of the specular multipath channel to the multiple-access channel and obtain arbitrarily tight upper and lower bounds on the performance of a correlation receiver when random sequences are utilized in the system. The approach that we use involves the use of conditional probability density functions. This has special significance to packet radio systems because the analysis of such systems involves the consideration of conditional probability density functions.

The method that we develop is particularly useful because not only do we obtain arbitrarily tight bounds on the average probability of bit error of the system, but we also obtain two vectors that serve as bounds on the probability density function of the multiple-access interference. This allows us to study the nature of various approximations to the average probability of bit error which have been proposed. This also allows the approach to be applied to other methods of signaling and demodulation. Furthermore, the approach we describe involves vector operations, and computations can therefore be performed efficiently and inexpensively on an array processor.

This thesis consists of five chapters. In Chapter 2 of the thesis we describe a packet radio system and explain how the analysis of the packet radio system reduces to the consideration of a multiple-access system when the number of active transmitters is known. The specular multipath channel that is encountered in this multiple-access system is then described in more detail. Results on the performance of the multipath-combining receiver are developed in Chapter 3. Arbitrarily tight upper and lower bounds on the performance of a system involving the multiple-access channel, a correlation receiver, and random sequences are developed in Chapter 4. Finally, a summary and a conclusion of the thesis are given in Chapter 5.

CHAPTER 2

CHANNEL MODEL

2.1 Introduction

In this chapter we are concerned with the communication channel that links mobile radios in a communication network. Our approach is to describe first the overall packet radio system. We then describe the communication channel that exists when the number of transmitting radios in the overall packet radio system is known. This channel is defined by giving for each of the active transmitters an expression for the resulting input to a typical receiver. The channel parameters are random variables that model the combined effects of all the features of the environment. Finally, the statistics and dependencies of these random channel parameters are described.

2.2 Packet Radio Network

It is advantageous to define first the communication network. The network consists of a large number N_R of radios in a localized geographical region. Most of the time any particular radio is not transmitting. We refer to a radio that is not transmitting as an inactive (or silent) radio, and a radio that is transmitting as an active radio. An active radio transmits a burst of digital data called a packet. For example, a packet might consist of a thousand data bits which are grouped together and sent consecutively.

The number of active radios in the network is a random variable κ. If we denote the probability that the k-th radio is active by $p_a(k)$, the expected number of active radios in the network is given by

$$\bar{\kappa}=\sum_{k=1}^{N_R} p_a(k). \tag{2.1}$$

It may be that each radio in the network is active with the same probability p_a, independently of all other radios in the network. In this case, the number of active radios κ is a binomial random variable with parameters N_R and p_a. For certain values of N_R and p_a, the random variable κ is accurately modeled by a Poisson random variable with mean $\bar{\kappa}=N_R p_a$ [13]. Although we define the communication channel for a fixed number K of active transmitters, it is important to remember that the number of active transmitters κ is itself a random quantity.

If we can determine the average performance of a typical receiver for each value of the discrete random variable κ, we can obtain the overall average performance by averaging with respect to the random variable κ. Suppose that we can determine the average probability of error of a typical receiver for any number of active radios in the network. We denote the average probability of error when n transmitters are active by $P_{avg}(n)$. The overall average probability of error P_{avg} is given by

$$P_{avg}=\sum_{n=1}^{\infty} p_{\kappa}(n) P_{avg}(n), \tag{2.2}$$

where $p_{\kappa}(n)$ is the discrete density function of the random variable κ.

2.3 Multiple-Access Communication System Which Exists When K Radios Are Active

In the remainder of this chapter we assume that there are K active transmitters, i.e., we define the communication channel conditioned on the event that the random variable κ equals K. The system we are considering can now be viewed as a multiple-access communication system with K users. Our present goal is to describe the communication channel which exists when there are K active transmitters and a single typical receiver. One of the K active radios transmits information intended for the receiver, and the other K-1 active radios produce undesirable multiple-access interference.

It is important to realize that the performance of a typical receiver can vary widely even though the number K of active radios in the network is fixed. If the interfering radios all happen to be close to the receiver, the multiple-access interference can be very large. Correspondingly, if the interfering radios all happen to be far from the receiver, the multiple-access interference can be very small. Many other features of the geography also influence the characteristics of the communication channel. The geographical features that determine the characteristics of a typical link change as radios change location. In our model of a typical link, the random link parameters model the net effect of all of the complex and changing geographical features that determine the communication channel properties when a known number K of radios are active. The average performance on the typical link indicates the average performance on all other links in the system.

2.4 Communication Channel Model

We are now in a position to specify the properties of the communication channel. Suppose we wish to analyze the performance of the i-th receiver. The relevant properties of the communication channel are specified by giving for each of the K active transmitters an expression for the resulting input to the i-th receiver. This received signal from a typical transmitter consists of a random number of replicas of the transmitted signal. The delay, amplitude, and phase associated with each replica are also random.

2.4.1 Analytic Signals

Our approach is to specify real signals by analytic signals which are defined in [14]. The analytic signal corresponding to a real signal x(t) is given by

$$z(t)=x(t) + j\hat{x}(t), \tag{2.3}$$

where $\hat{x}(t)$ denotes the Hilbert transform of x(t). The complex signal z(t) has only positive frequency components. The Fourier Transform of z(t) is given by

$$Z(\omega) = \begin{cases} 2X(\omega) & \text{if } \omega \geq 0 \\ 0 & \text{if } \omega < 0 , \end{cases} \tag{2.4}$$

where $X(\omega)$ is the Fourier Transform of the real signal $x(t)$. The analytic signals allow us to specify conveniently both the amplitude and phase of complicated signals, and simplify the description and analysis of complex receiver structures that will be encountered later.

2.4.2 Transmitted Bandpass Signal

Since the signals encountered are bandpass signals, it is convenient to specify the signal transmitted by the k-th transmitter by $Re[z_k(t)]$, where

$$z_k(t) = \sigma_k(t)\exp(j\omega_0 t). \tag{2.5}$$

The parameter ω_0 is the carrier frequency of the bandpass signal of (2.5). The complex signal $\sigma_k(t)$ is a baseband signal component that must be band limited to the frequency interval $[-\omega_0,\infty)$ in order for $z_k(t)$ to be an analytic signal. In a direct-sequence spread-spectrum multiple-access (DS/SSMA) communication system $\sigma_k(t)$ represents the modulation, the spectral-spreading waveform, and the carrier phase. We assume there are K active radios in the communication network. Hence, k is an element of the set $\{1,\ldots,K\}$.

2.4.3 Received Bandpass Signal

Several studies of the communication channel encountered in a mobile radio environment have been performed for a variety of terrain conditions [15]-[17]. In [17], measurements that were made using a spread-spectrum waveform centered at 1370 MHz are described. In these measurements the transmitter was elevated and fixed while the receiver was mobile. The spread-spectrum waveform was a direct-sequence waveform, having either 10 or 20 MHz of bandwidth. This allowed an accurate resolution of time delays.

Measurements showed that whether the terrain was a dense urban terrain or a modest urban terrain with a few tall buildings, the received signal consisted of several resolvable signal replicas with varying delays. The maximum (differential) delay of any received signal replica was about 6 μs. This delay translates to an excess path length of about one mile. Also, reflections appeared to come from objects no farther from the receiver than about one mile.

As in [12] and [18], we assume that the k-th transmitted signal, which is given by $\mathrm{Re}[z_k(t)]$, results in a signal at the i-th receiver which is given for $k \neq i$ by

$$\mathrm{Re}[p(k,i;t)\exp(j\omega_0 t)] + n(t), \tag{2.6}$$

where

$$p(k,i;t) = \sum_{\lambda=1}^{L(k,i)} g(k,i;\lambda)\sigma_k(t-\tau(k,i;\lambda))\exp[j\theta(k,i;\lambda)]. \tag{2.7}$$

Since there are K radios in the network, the indices k and i are elements of the set $\{1,\ldots,K\}$. In (2.7), the random variable $L(k,i)$ represents the number of replicas of the signal of transmitter k that are present at receiver i. Alternatively, the random variable $L(k,i)$ represents the number of signal paths that exist from the k-th transmitter to the i-th receiver. The random variables $g(k,i;\lambda)$, $\tau(k,i;\lambda)$, and $\theta(k,i;\lambda)$ represent the amplitude, delay, and carrier phase associated with the λ-th replica at receiver i of a signal from transmitter k. Since there are $L(k,i)$ received replicas, λ is an element of the set $\{1,\ldots,L(k,i)\}$. The term $n(t)$ in (2.6) represents additive white Gaussian noise (AWGN) and models the thermal noise at the receiver.

It is convenient to define several other parameters in order to simplify the notation that will follow. We define $L=L(i,i)$, $g_\lambda=g(i,i;\lambda)$, $\tau_\lambda=\tau(i,i;\lambda)$, $\theta_\lambda=\theta(i,i;\lambda)$, $\phi_\lambda=\theta_\lambda-\omega_0\tau_\lambda$, $\tau_{k,\lambda}=\tau_k-\tau_\lambda$, and $\phi_{k,\lambda}=\phi_k-\phi_\lambda$. We also define the parameters $\epsilon_{k,i}$ by

$$\epsilon_{k,i} = \sum_{\lambda=1}^{L(k,i)} g^2(k,i;\lambda). \tag{2.8}$$

For convenience, we define $\epsilon = \epsilon_{i,i}$.

2.4.4 Statistical Properties of Channel Parameters

The relationships among the random variables representing the number of signal paths from a transmitter to a receiver, as well as the amplitudes, delays, and carrier phases associated with the paths have been studied at frequencies around 1280 MHz and described in [12] and [15]. These relationships are summarized in [12]. The characteristics of the channel for closely spaced geographical points are dependent. Also, the characteristics of two signal replicas from the same transmitter with similar delays are dependent. Experimental studies have shown that signal replicas with small associated delays are more likely than those with larger delays. In fact, the probability of a signal replica of significant amplitude arriving with a (differential) delay greater than about 6-7 μs is very small.

We will at times consider a simplified model of the real communication system in which we assume the properties of the links from each of the K active transmitters to the listening receiver are mutually independent. Furthermore, we will assume that the random parameters characterizing a set of signal replicas arriving at the listening receiver from the same transmitter form a mutually independent set of random variables if no two replicas have associated delays that are within a chip duration of one another. (A chip duration is a small increment of time that will be defined in Chapter 3 when direct-sequence spread-spectrum modulation is discussed.) In the notation of Section 2.4.3, the random variables $\theta(k,i;\lambda)$, $g(k,i;\lambda)$, $\tau(k,i;\lambda)$, and $L(k,i)$, together with the data symbols, are assumed to form a set of mutually independent random variables. Also, each phase $\theta(k,i;\lambda)$ is assumed to be uniformly distributed on the interval $[0,2\pi)$.

Although the arrival of transmitted signal replicas with similar associated delays at a receiver are found to be somewhat dependent events in these experimental studies, we will at times consider a simpler model in which the arrivals are assumed to be independent events. We assume we have experimentally measured a path delay density function $P_D(x)$ which vanishes outside the interval $[0,\Delta]$, where Δ is the maximum path delay. The probability of a signal replica arriving in an infinitesimal interval $[x, x+dx]$ is then given by $P_D(x)dx$, and the expected number of replicas at a receiver is given by

$$\overline{L} = \int_0^{\Delta} P_D(x)dx \, . \tag{2.9}$$

The arrivals can now be modeled by a nonhomogeneous Poisson process with parameter $P_D(x)$ [19].

CHAPTER 3

MULTIPATH DIVERSITY RECEPTION OF SPREAD-SPECTRUM COMMUNICATIONS

3.1 Introduction

In this chapter we are concerned with the reception of data through the specular multipath channel that has been described in Chapter 2. Previous analyses (e.g., [10] and [11]) of direct-sequence spread-spectrum communications through a specular multipath channel involve a single user, biphase modulation, and a correlation receiver. In this chapter we are interested in systems which involve multiple transmitters and receivers and which utilize more general modulation techniques. We are also interested in receiver structures that are more complex than the simple correlation receiver. The simple correlation receiver extracts the information content of just one of the multiple replicas of the transmitted signal which are present at the input of the receiver. The remaining replicas simply produce undesirable interference. The more complex receiver that we consider extracts information from all the received signal replicas. Therefore, the performance of the complex receiver is considerably better than the performance of the simple correlation receiver.

In general, the optimum receiver for direct-sequence spread-spectrum signaling through the specular multipath channel is unknown. However, if the receiver has knowledge of the channel parameters, no multiple-access interference is present, and white Gaussian noise is added at the receiver, then the optimum receiver for the detection of a single data pulse is matched to the signal that is actually received from the transmitter. In the systems which we are considering, these conditions are not met. First of all, we are interested in the reception of a sequence of successive data pulses instead of a single data pulse. When successive data pulses are received, the effects of intersymbol interference can be severe because there may be channel path delays that are greater than the data pulse duration. Transmissions from other terminals are an additional source of noise. Also, the channel parameters must generally be estimated and are therefore subject to error.

Nevertheless, a receiver that is matched to the signal actually received from the transmitter remains of interest as approximately optimal when direct-sequence spread-spectrum signaling is employed. Direct-sequence spread-spectrum signaling combats the intersymbol interference and provides multiple-access capability. In fact, as the length of the signature sequences that are used in the system is increased, the intersymbol interference and the multiple-access interference become smaller. Therefore, the conditions for optimality of a receiver that is matched to the signal actually received from the transmitter are more nearly met as the length of the signature sequences grows.

In this chapter we analyze the performance of a receiver, related to one described in [12], that is matched to the received signal from a particular transmitter. The present analysis considers general quadriphase modulation and accounts for both intersymbol interference and multiple-access interference. It yields expressions for the signal-to-noise ratio and approximations to the average probability of bit error in terms of parameters of the signature sequences. The results identify the parameters that are important in selecting signature sequences for the system. They also quantify the multiple-access capability that exists when the communication system employs a multipath-combining receiver.

In a practical communication system the receiver does not have perfect knowledge of the parameters of the specular multipath channel. Although our results show that the achievable performance gains are very large, we expect a practical receiver to perform worse than the ideal receiver that we analyze. Our results show that performance gains are possible. The extent to which these gains are achieved in a practical communication system depends on the accuracy with which the receiver can estimate the channel parameters.

3.2 System Model

We are interested in a system that employs a modulation related to the direct-sequence quaternary modulation described in [3]. An understanding of the analyses of the systems given in [2] and [3] is a helpful aid in examining the following analysis. In the signaling scheme that we are considering, the transmitter consists of two branches. In each branch a binary data waveform with bit duration T is multiplied by a spectral-spreading signal, which encodes the data and expands the bandwidth that the data signal occupies. The two encoded binary data waveforms together modulate a carrier producing a quaternary signaling sheme. The signaling model that we consider is general enough to include binary-phase-shift-keying (BPSK), quadriphase-shift-keying (QPSK), offset quadriphase-shift-keying (OQPSK), and minimum-shift-keying (MSK).

A key difference between the present system (including transmitter, channel, and receiver) and the system described in [3] is in the channel. In the present system the channel linking a transmitter to the intended receiver yields an output from each bit that lasts up to $T+\Delta$ seconds, where Δ is the maximum delay that is associated with any received signal replica [10]-[12]. As mentioned in Chapter 2, the parameter Δ may be several times larger than the data bit duration. This means that it is possible for the difference between the delays of a pair of signal replicas to be a multiple of the data bit duration. If this happens, two signal replicas corresponding to the transmission of two different bits arrive at the receiver simultaneously. If the same signature sequence is used for the transmission of successive bits, the problem of intersymbol interference can be very severe. If the system is designed such that signal replicas corresponding to different bits never arrive at the receiver simultaneously unless they are coded with different signature sequences, the problem of intersymbol interference is less severe.

In order to discriminate between the received signal replicas that result from the transmission of a series of successive bits, we assign each transmitter two sequences of length p. One is assigned to the in-phase component of the transmitted signal, and the other is assigned to the quadrature component of the transmitted signal. Each sequence of length p consists of s subsequences of length N (i.e., $p=sN$). The s subsequences are used sequentially for the transmission of s data bits. They are then used again in the same order for the transmission of s more data bits. We choose $s=\lfloor \Delta/T+1/N \rfloor$ so that two signal replicas corresponding to different data bits never arrive at the receiver within a chip duration (this small time interval will be defined in the following) of one another unless they are coded with different signature sequences.

3.2.1 Transmitter Model

In this section we define the properties of the transmitted signals in this quaternary spread-spectrum system. If we focus on just one receiver section of the quaternary spread-spectrum system, we obtain the corresponding description of a binary spread-spectrum system.

The k-th transmitter generates a pair of data signals $b_{2k-1}(t)$ and $b_{2k}(t)$ that are given by

$$b_n(t) = \sum_{\lambda=-\infty}^{\infty} b_\lambda^{(n)} p_T(t-\lambda T) \tag{3.1}$$

for $n=2k-1$ and $n=2k$, where the unit rectangular pulse $p_T(t)$ is given by $p_T(t) = 1$ for $0 \leqslant t < T$, and $p_T(t) = 0$, otherwise. The data symbols $b_\lambda^{(n)}$ are independent and identically distributed random variables for which $\Pr\{b_\lambda^{(n)} = +1\} = \Pr\{b_\lambda^{(n)} = -1\} = 1/2$. An even integer n corresponds to an in-phase signal component, and an odd integer n corresponds to a quadrature signal component.

The k-th transmitter is assigned two binary sequences $a^{(2k-1)}$ and $a^{(2k)}$ for the quadrature channel and the in-phase channel, respectively. The sequence (a_n) is used to form the spectral-spreading signal $a_n(t)$ that is given by

$$a_n(t) = \sum_{j=-\infty}^{\infty} a_j^{(n)} \psi(t-jT_c), \tag{3.2}$$

where $\psi(t)$ is the common chip waveform for all signals. The chip waveform $\psi(t)$ is time limited to the interval $[0,T_c]$, where the chip duration T_c satisfies the relationship $T_c = T/N$. The chip waveform is also normalized such that $\int_0^{T_c} \psi^2(t)dt = T_c$. Each signature sequence (a_n) has period p, where $p=sN$ and s is an integer. Hence, during one period of the spectral spreading signal, s signature signals corresponding to the s signature subsequences are used sequentially for the transmission of s data bits.

We now specify the transmitted signals in terms of the analytic signal representation described in Chapter 2. The k-th transmitter sends $\mathrm{Re}[z_k(t-T_k)]$, where

$$z_k(t) = \sigma_k(t)\exp(j\omega_0 t) \tag{3.3}$$

and

$$\sigma_k(t) = [b_{2k}(t-t_0)a_{2k}(t-t_0) - jb_{2k-1}(t)a_{2k-1}(t)]\exp(j\alpha_k). \tag{3.4}$$

In the above expressions, α_k is the carrier phase of the k-th transmitter, t_0 is a delay parameter that is necessary for the generation of MSK and OQPSK, and T_k is a random variable that is needed to model the asynchronous system. Each transmitted signal undergoes a random phase shift, modeled by the random variables $\theta(k,i;\lambda)$ for λ in the set $\{1,\ldots,L(k,i)\}$, when it passes through the communication channel. Since only phase shifts modulo 2π are of interest, we assume $\alpha_k = 0$ for $1 \leqslant k \leqslant K$ without losing any generality. Since only relative time delays are of importance and we are studying the i-th receiver, we set $T_i = 0$. Because of the structure of the transmitted signals and the stationarity of the thermal noise that is added at the receiver, the time delays T_k are only important modulo sT. We model each time delay T_k, for $k \neq i$, as a random variable that is uniformly distributed on the interval $[0,sT)$ and independent of all other random variables that have been defined in the system and channel models. We choose the offset parameter $t_0 = T_c/2$. The choice of this offset parameter is discussed in [3].

3.2.2 Receiver Model

The overall structure of the receiver is shown in Figure 3.1 for the case in which five signature subsequences are used sequentially for the transmission of data, i.e., for the case s=5. The structures in Figure 3.1 which are denoted by $\mathcal{B}(n,m)$ are defined in more detail in Figure 3.2.

The receiver consists of an in-phase section and a quadrature section. Each section consists of s branches. In the m-th branch a filter matched to the m-th signature signal is followed by a transversal filter. When a bit is coded with the m-th signature signal and transmitted, the filter matched to that

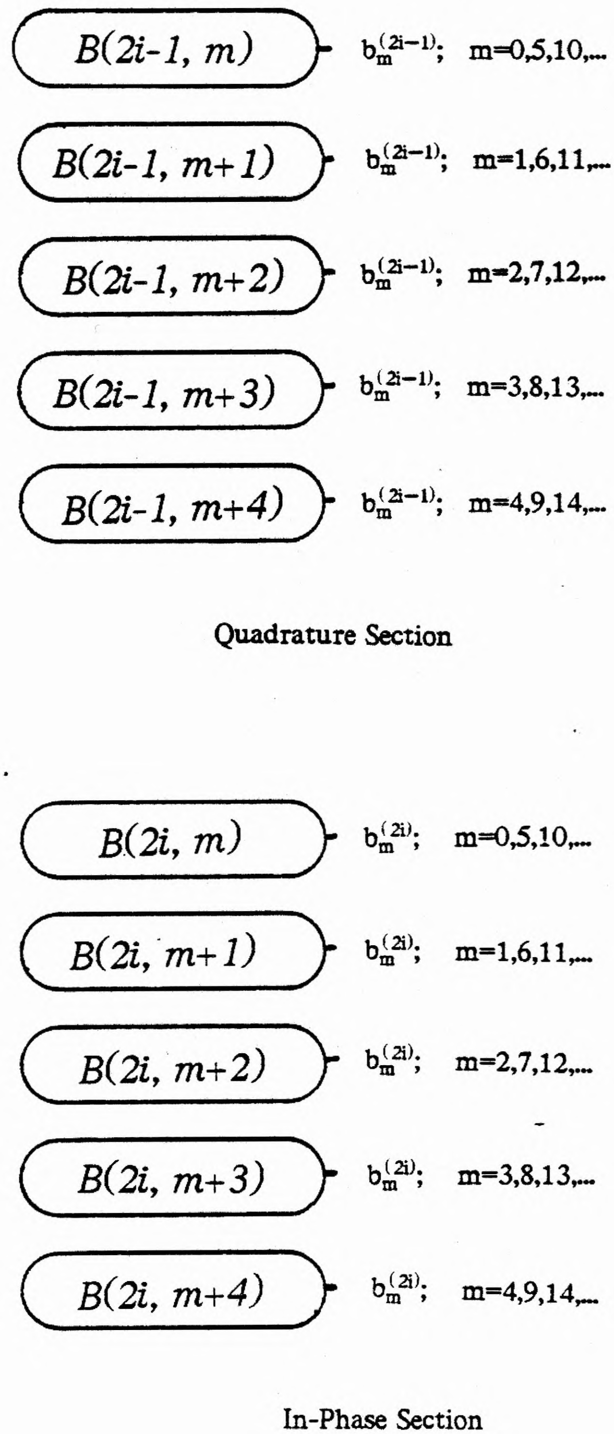

Figure 3.1. Overall structure of the i-th receiver (s=5).

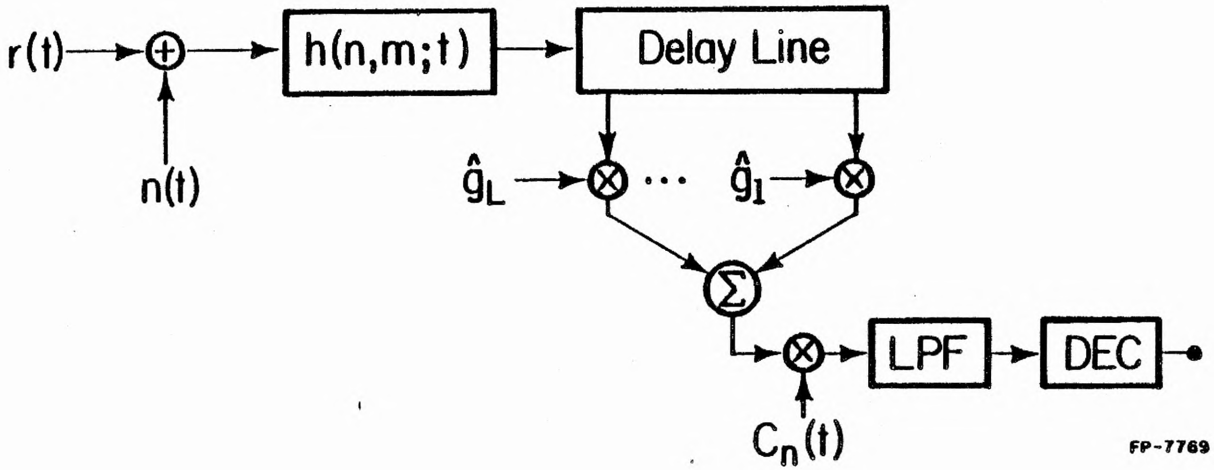

Figure 3.2. General branch of the receiver, $B(n,m)$.

signal will yield output peaks corresponding to each channel path during an interval of width less than sT. The transversal filter coherently combines the output peaks. Bits that are coded with other than the m-th signature signal do not produce large peaks in the m-th branch if the subsequences are properly chosen.

Each branch yields a bit decision every sT seconds, but the outputs from the s branches are staggered by T seconds. The output stream is taken sequentially from the branches of a receiver section to produce an output bit every T seconds.

A general receiver branch is shown in Figure 3.2. The channel output is denoted by r(t), and the white Gaussian noise process is denoted by n(t). The parameter n specifies a particular section of a receiver. For the i-th receiver we are concerned with n=2i-1 and n=2i, and in general n is an element of the set {1,...,2K}. The parameter m specifies a particular branch of a receiver section, and in general m is an element of the set {0,...,s−1}. The m-th branch of section n is matched to the m-th signature signal used by transmitter section n. The impulse response of the matched filter in the m-th branch of section n is given by

$$h(n,m;t) = 2\mathrm{Re}\left[\sum_{\lambda=0}^{N-1} a_{\lambda+Nm}^{(n)} \psi(T-t-\lambda T_c)\exp(j\omega_0 t)\right]. \tag{3.5}$$

The transversal filter is supplied information about the channel parameters. When a signal Re[z(t)] is applied to its input, the output is given by

$$\mathrm{Re}\left[\sum_{\lambda=1}^{L} g_\lambda \exp(-j\theta_\lambda) z(t-sT+\tau_\lambda)\right], \tag{3.6}$$

where g_λ and θ_λ are defined as in Chapter 2. In Figure 3.2, $g_\lambda \exp(-j\theta_\lambda)$ is denoted by $\hat{g}_\lambda$.

For detection of an in-phase signal, corresponding to an even value of n, the output of the transversal filter is multiplied by $2\cos[\omega_0(t-sT)]$. For detection of a quadrature signal, corresponding to an odd value of n, the output of the transversal filter is multiplied by $2\sin[\omega_0(t-sT)]$. In Figure 3.2, the appropriate signal is denoted by $c_n(t)$. The resulting product is passed through a low-pass filter, sampled, and compared with a threshold to make a decision on the bit $b^{(n)}_{ks+m}$, where k is an integer. The sampling times are at $t=ksT+t_m$ if n is even, and $ksT+t_m-t_0$ if n is odd, where $t_m=(s+m+1)T+t_0$ and k is an integer.

3.3 Correlation Functions

In this section we introduce the correlation functions needed for the analysis that follows. These functions are related to some of the correlation functions defined in [3]. However, because the receiver we consider combines information during an interval of several bit durations, it is necessary to define some new correlation functions in order to simplify the following notation. After defining the correlation functions, we state convenient ways of evaluating these functions and explore their dependence on the signature subsequences used in the system.

The correlation function defined by

$$C(k,i;\lambda)(t) = \int_0^T b_k(x+\lambda T+t)a_k(x+\lambda T+t)a_i(x+\lambda T)dx, \tag{3.7}$$

where k and i are elements of the set $\{1,\ldots,2K\}$ and λ is an integer, is useful for denoting the correlation of the λ-th signature signal of the i-th receiver section with the data-modulated spreading signal of the k-th transmitter section. When the correlation function is used for this purpose, the parameter t specifies the interval of the data-modulated spreading signal used in performing the correlation. Since in the following analysis we are interested in two particular expressions that involve these functions, for convenience we define

$$F(i,\lambda)(t) = C(i,i;\lambda)(t) + C(i,i,\lambda)(-t) \tag{3.8}$$

and

$$G(i,\lambda)(t) = C(i+1,i,\lambda)(t+t_0) - C(i+1,i,\lambda)(-t+t_0). \tag{3.9}$$

The correlation function defined by

$$C(k,i;m,n)(t) = \int_0^T a_k(x-t+mT)p_T(x-t)a_i(x+nT)dx, \tag{3.10}$$

where k and i are elements of the set $\{1,\ldots,2K\}$ and m and n are integers, is useful in denoting the correlation of the m-th signature signal of transmitter section k with the n-th signature signal of receiver section i. The parameter t specifies the offset of the two signals. Notice that the correlation function $C(k,i;m,n)(t)$ is equal to zero for $|t|>T$.

For $\lambda = \lfloor t/T_c \rfloor$,

$$C(k,i;m,n)(t) = C(k,i;m,n)(\lambda T_c)\hat{R}_\psi(s) + C(k,i;m,n)[(\lambda+1)T_c]R_\psi(s), \tag{3.11}$$

where $s=t-\lambda T_c$ and $\hat{R}_\psi(s)$ and $R_\psi(s)$ are the continuous-time partial autocorrelation functions for the chip waveform [3]. The continuous-time partial autocorrelation functions of the chip waveform are given for $0 \leqslant s \leqslant T_c$ by

$$R_\psi(s) = \int_0^s \psi(t)\psi(t+T_c-s)dt \tag{3.12}$$

and

$$\hat{R}_\psi(s) = \int_s^{T_c} \psi(t)\psi(t-s)dt. \tag{3.13}$$

For $s>T_c$ or $s<0$, the functions $R_\psi(s)$ and $\hat{R}_\psi(s)$ are defined to be zero. Notice that $R_\psi(s) = \hat{R}_\psi(T_c-s)$ for $0 \leqslant s \leqslant T_c$.

The correlation function $C(k,i;m,n)(t)$ depends on the chip waveform through the function $R_\psi(s)$ and on the appropriate signature sequences through the function $C(k,i;m,n)(\lambda T_c)$. In fact, the function $C(k,i;m,n)(\lambda T_c)$ for λ in the set of integers is the discrete aperiodic correlation function, which is defined in [3], for the m–th signature subsequence of transmitter section k and the n-th signature subsequence of receiver section i when m and n are elements of the set $\{0,\ldots,s-1\}$.

In examining the effects of multiple-access interference, we are interested in the parameter given by

$$\sigma^2(k,i;m,n) = \tfrac{1}{2}T^{-3}\int_{-\infty}^{\infty} C(k,k;m,m)(x)C(i,i;n,n)(x)dx\,. \tag{3.14}$$

By applying (3.11) to (3.14), we find that this parameter can be expressed as

$$\sigma^2(k,i;m,n) = T^{-3}[\mu(k,i;m,n)(0)M_\psi + \mu(k,i;m,n)(1)M'_\psi]\,, \tag{3.15}$$

where

$$\mu(k,i;m,n)(L) = \sum_{\lambda=1-N}^{N-1} C(k,k;m,m)(\lambda T_c)C(i,i;n,n)[(\lambda+L)T_c]\,, \tag{3.16}$$

$$M_\psi = \int_0^{T_c} R_\psi^2(s)ds\,, \tag{3.17}$$

and

$$M'_\psi = \int_0^{T_c} R_\psi(s)\hat{R}_\psi(s)ds. \tag{3.18}$$

For convenience we define

$$\mu(k,i;n)(L) = \frac{1}{s}\sum_{m=0}^{s-1}\mu(k,i;m,n)(L)\,. \tag{3.19}$$

In the analysis of the effects of intersymbol interference, we are concerned with computing the function $C^2(k,i;m,n)(t)$ for various values of t. The function $C^2(k,i;m,n)(t)$ can be expressed using (3.11). If the m-th sequence of section k is a random sequence (i.e., a sequence of independent and identically distributed random variables x_j for which $\Pr\{x_j = +1\} = \Pr\{x_j = -1\} = 1/2$) and if $|\lambda| \leqslant N$, it follows that

$$E\{C^2(k,i;m,n)(\lambda T_c)\} = N - |\lambda| \tag{3.20}$$

and

$$E\{C(k,i;m,n)(\lambda T_c)C(k,i;m,n)[(\lambda+1)T_c]\} = \begin{cases} \sum_{j=\lambda}^{N-2} a_{nN+j}^{(i)}\, a_{nN+j+1}^{(i)} & ,\ \lambda \geqslant 0 \\ \sum_{j=0}^{N-1+\lambda} a_{nN+j}^{(i)}\, a_{nN+j+1}^{(i)} & ,\ \lambda < 0\,. \end{cases} \tag{3.21}$$

Equations (3.11), (3.20), and (3.21) can be used to evaluate the function $E\{C^2(k,i;m,n)(t)\}$ when the m-th subsequence of section k is a random sequence. If the n-th sequence of section i is also random, the right side of (3.21) is zero.

3.4 Analysis of a General Receiver Branch

In this section we obtain the signal-to-noise ratio at the output of a general receiver branch. We also obtain approximations to the average probability of bit error of the receiver. We focus on the m-th branch of the in-phase section of the i-th receiver (i.e., n=2i). The analysis of the quadrature section of the receiver is similar. At the sampling time we compute the variance of the random input to the decision circuit. The variance consists of four components. These are a result of the random path strengths, intersymbol interference, thermal noise at the receiver, and multiple-access interference. In one approximation to the average probability of bit error, we model the total random input to the decision circuit by a Gaussian random variable. In another approximation, we consider the random input to the decision circuit conditioned on a set of parameters of the system. We then approximate the random input to the decision circuit by a Gaussian random variable. We next average the resulting expression for the conditional average probability of bit error with respect to the appropriate random parameters of the system in order to obtain an expression for the overall average probability of bit error.

3.4.1 Response to the Desired Transmission

The i-th transmitted signal has been defined in (3.3) and (3.4), the specular multipath channel has been defined in (2.6) and (2.7), and the in-phase branch of the i-th receiver has been defined in (3.5) and (3.6). Since the receiver is a linear filter, we can combine this information in order to obtain an expression for the output which results from the desired (i.e., the i-th) transmission. This output from the transversal filter of the in-phase branch of the i-th receiver is given by

$$y_{MT}(t) = \mathrm{Re}\{\gamma_0(t)\exp[j\omega_0(t-sT)]\}, \tag{3.22}$$

where

$$\gamma_0(t)=\sum_{k=1}^{L}\sum_{\lambda=1}^{L} g_k g_\lambda\{\exp(j\phi_{k,\lambda})C(n,n,m)(t-t_m+\tau_{\lambda,k}) + \exp[j(\phi_{k,\lambda}-\tfrac{\pi}{2})]C(n+1,n,m)(t-t_m+\tau_{\lambda,k}+t_0)\} \tag{3.23}$$

The output is multiplied by a synchronous signal, filtered by an ideal low-pass filter, and applied to the decision circuit (see Figure 3.1 and Figure 3.2). By using (3.7), (3.8), and (3.9) in (3.22) and (3.23) and appropriately combining terms, we find that the input to the decision circuit can be expressed at time t_m as

$$y_0(t_m) = b_m^{(n)}\epsilon T + \sum_{k=1}^{L}\sum_{\lambda=k+1}^{L} g_k g_\lambda[\cos\phi_{k,\lambda}F(n,m)(\tau_{\lambda,k}) + \sin\phi_{k,\lambda}G(n,m)(\tau_{\lambda,k})]. \tag{3.24}$$

When there is only one channel path, the receiver reduces to a correlation receiver, which is analyzed in [11]. We specialize to the case in which there are at least two channel paths. In the following analysis we are concerned with expectations conditioned on the event that there are at least two channel paths. We denote this conditional expectation by $E'\{\cdot\}$. A variance involving this conditional expectation is denoted by $Var'\{\cdot\}$.

Using (3.24) and evaluating the expected value of $y_0(t_m)$ with respect to the random path strengths and phases, we find (in the notation of Chapter 2)

$$E'\{y_0(t_m)\} = b_m^{(n)} T E'\{\epsilon\}. \tag{3.25}$$

If $E\{g_k^2\} = G$ for each positive integer k, (3.25) reduces to

$$E'\{y_0(t_m)\} = b_m^{(n)} TGE'\{L\}. \tag{3.26}$$

For most cases of interest, $\overline{L}$ is large. This implies $E'\{L\} \approx \overline{L}$, where $\overline{L}$ denotes the expected value of the random variable L.

The variance of $y_0(t_m)$ is determined from (3.24) and (3.25) by first taking the expectation with respect to the random phases. We find that

$$Var'\{y_0(t_m)\} = N_s + N_I, \tag{3.27}$$

where

$$N_s = T^2 Var'(\epsilon) \tag{3.28}$$

and

$$N_I = \frac{1}{2} E'\left\{ \sum_{k=1}^{L} \sum_{\lambda=k+1}^{L} g_k^2 g_\lambda^2 \left[F^2(n,m)(\tau_{\lambda,k}) + G^2(n,m)(\tau_{\lambda,k})\right] \right\}. \tag{3.29}$$

If $E\{g_k^2\} = G$ and $Var\{g_k^2\} = V_p$ for each positive integer k, (3.28) reduces to

$$N_s = T^2\,[G^2\,Var'\{L\} + V_p E'\{L\}]. \tag{3.30}$$

For most cases of interest, $Var'\{L\} \approx Var\{L\} = \overline{L}$.

The expectation in (3.29) is meant to be taken with respect to the path delays, the path strengths, and the data symbols. Assuming all delay differences are greater than t_0, evaluating this expectation with respect to the data symbols and path strengths gives

$$N_I = \frac{1}{2}E'\left\{\sum_{k=1}^{L}\sum_{\substack{\lambda=1\\ \lambda\neq k}}^{L} E\{g_k^2\}E\{g_\lambda^2\}V(n,m)(\tau_{\lambda,k})\right\}, \tag{3.31}$$

where

$$V(n,m)(t) = \sum_{\lambda=-\infty}^{\infty}\{C^2(n,n;m+\lambda,m)(\lambda T-t) + C^2(n+1,n;m+\lambda,m)(\lambda T-t-t_0)\} \tag{3.32}$$

$$-C(n+1,n;m,m)(-t_0+t)C(n+1,n;m,m)(-t_0-t) + C^2(n,n;m,m)(t).$$

This can be written as

$$N_I = \int_{-\infty}^{\infty} D(x)V(n,m)(x)dx, \tag{3.33}$$

where $D(x)$ is a function which depends on the channel statistics. The function $D(x)$ is given for $|x| \geq T_c$ by

$$D(x) = \frac{1}{2}E'\left\{\sum_{k=1}^{L}\sum_{\substack{\lambda=1\\ \lambda\neq k}}^{L} E\{g_k^2\}E\{g_\lambda^2\}P_{k,\lambda}(x)\right\}, \tag{3.34}$$

where $P_{k,\lambda}(x)$ is the probability density function of the difference of the k-th and λ-th path delays (i.e., of $\tau_{k,\lambda}$). For $|x|<T_c$, we define D(x) to be zero. This approximation is justified since the condition $|x|<T_c$ corresponds to the situation in which two signal replicas arrive at the receiver with a delay difference of less than the chip duration T_c. The probability of this event is small if $\Delta/T_c >> \overline{L}$. Furthermore, if the event does occur, we assume the receiver treats these two paths as a single path.

For the channel model which we consider, (3.34) reduces for $|t| \geq T_c$ to

$$D(x) = \frac{1}{2} G^2 P_D(x) * P_D(-x)[1-(\overline{L}+1)\exp(-\overline{L})]^{-1}, \tag{3.35}$$

where the asterisk denotes a convolution and we have assumed $E\{g_k^2\} = G$ for each positive integer k.

The results of Section 3.3 can be used to express N_I in terms of the sequences used by the i-th transmitter. If we assume D(x) is approximately constant during a chip duration, we can use (3.11), (3.17), and (3.18) in (3.33) in order to express the integral as a discrete sum. If random sequences are used, the discrete sum which results can be reduced further by using (3.20) and (3.21).

3.4.2 Thermal Noise

If a stationary noise process at the output of the matched filter is given by $\mathrm{Re}[n_m(t)]$, the corresponding output from the transversal filter is given by the real part of the complex signal

$$n_{MT}(t) = \sum_{\lambda=1}^{L} g_\lambda n_m(t+\tau_\lambda - sT)\exp(-j\theta_\lambda). \tag{3.36}$$

The autocorrelation function of $n_{MT}(t)$ is given [14] by

$$R_N(\tau) = \frac{1}{2} E'\{n_{MT}(t+\tau) n_{MT}^*(t)\} \;\; = \frac{1}{2} E'\{\epsilon\} E\{n_m(t+\tau) n_m^*(t)\}. \tag{3.37}$$

If the noise input to the matched filter is white with two-sided spectral density $N_0/2$, (3.37) can be written [14] as

$$R_N(\tau) = \frac{N_0}{4}E'\{\epsilon\}h(n,m;\tau) * h^*(n,m;-\tau), \tag{3.38}$$

where the asterisk which is not a superscript denotes a convolution. From (3.38), we can evaluate the variance of the thermal noise at the input to the decision circuit. This variance is given [14] by

$$N_T = R_N(0) = N_0TE'\{\epsilon\}. \tag{3.39}$$

If $E\{g_k^2\} = G$ for each positive integer k, (3.39) becomes

$$N_T = E'\{L\}GN_0T. \tag{3.40}$$

3.4.3 Multiple-Access Interference

We wish to evaluate the variance of the multiple-access noise at the output of the receiver. Our approach is to evaluate the autocorrelation function of the random process which models the multiple-access interference and to obtain the variance of the multiple-access noise from this autocorrelation function.

Since the receiver branch is a linear time-invariant structure during the demodulation of a data bit, the order of the transversal filter and the matched filter can be interchanged in the receiver branch, and the output signal will remain the same. For the evaluation of the variance of the multiple-access noise, we assume for convenience in analysis that the first filter in the cascade is the transversal filter. Hence, the output from the transversal filter that results from the k-th transmitter is the real part of the signal given by

$$y_T(k,i;t) = \sum_{m=1}^{L(k,i)} \sum_{n=1}^{L} g(k,i;m) g_n z_k(t - T_k - sT + \tau_n - \tau(k,i;m)) \exp[j(\theta(k,i;m) - \theta_n)], \tag{3.41}$$

where $z_k(t)$ is the transmitted signal defined in (3.3). The autocorrelation function of this signal can be written as

$$R_T(k,i;t,\tau) = \frac{1}{2} E'\{y_T(k,i;t+\tau) y_T^*(k,i;t)\}$$

$$= E'\left\{ \sum_{m=1}^{L(k,i)} \sum_{n=1}^{L} g^2(k,i;m) g_n^2 R(k,t+\tau_n-\tau(k,i;m),\tau) \exp(j\omega_0\tau) \right\}, \tag{3.42}$$

where

$$R(k;t,\tau) = \frac{1}{2} E\{\sigma_k(t+\tau-T_k-sT) \sigma_k^*(t-T_k-sT)\}. \tag{3.43}$$

The expression for the autocorrelation function $R_T(k,i;t,\tau)$ can be simplified by first considering (3.43). We can expand (3.43) using (3.3) and (3.4) and evaluate the expectation which is indicated in (3.43). We find that for the asynchronous system which we consider, the autocorrelation function $R(k;t;\tau)$ does not depend on the variable t so that we can denote it by $R(k;\tau)$. We see that the signal of the k-th transmitter is a wide-sense stationary random process when the system is asynchronous. Furthermore, we find that the autocorrelation function of the wide-sense stationary random process that models the signal from the k-th transmitter is given by

$$R(k,\tau) = \frac{1}{2sT} \sum_{j=0}^{s-1} [C(2k,2k,j,j)(\tau) + C(2k-1,2k-1;j,j)(\tau)]. \tag{3.44}$$

When random sequences are employed, the autocorrelation function is given by

$$R(k;\tau) = \frac{1}{T_c} \int_{-\infty}^{\infty} \psi(\tau+x) \psi(x) dx. \tag{3.45}$$

We can further simplify the expression for the autocorrelation function $R_T(k,i;t,\tau)$ by recalling that the tap weights of the transversal filter remain approximately constant during the time interval during which a data bit is being detected. This means that we can evaluate the expectation in (3.42) and express the autocorrelation function of the output from the transversal filter that results from the k-th transmitter as

$$R_T(k,i;t,\tau) = R_T(k,i;\tau)\,, \qquad \text{for all } t$$

$$= \bar{\epsilon}_{k,i}E'\{\epsilon\}R(k;\tau)\exp(j\omega_0\tau)\,, \tag{3.46}$$

where $\bar{\epsilon}_{k,i}$ denotes the expected value of $\epsilon_{k,i}$.

The output of the transversal filter passes through a linear filter that has an impulse response given by h(n,m;t). Since the autocorrelation function of the input to the linear filter is given by (3.46), the autocorrelation function of the output from the cascade of the transversal filter and the filter matched to the transmitted signal is given by [14]

$$R_{MT}(k,n,m;\tau) = \tfrac{1}{4}\bar{\epsilon}_{k,i}E'\{\epsilon\}[R(k,\tau)\exp(j\omega_0\tau)] * h(n,m;\tau) * h^*(n,m;-\tau)\,. \tag{3.47}$$

By performing the convolution indicated in (3.47), we find that the variance of the noise from the k-th transmitter at the input to the decision circuit is

$$N_M(k) = R_{MT}(k,n,m;0) = \bar{\epsilon}_{k,i}E'\{\epsilon\}\int_{-T}^{T} R(k;x)C(n,n;m,m)(x)dx\,. \tag{3.48}$$

Substituting from (3.44) and using (3.14), we find that

$$N_M(k) = \bar{\epsilon}_{k,i}E'\{\epsilon\}\frac{T^2}{s}\sum_{\lambda=0}^{s-1}[\sigma^2(2k,n;\lambda,m) + \sigma^2(2k-1,n;\lambda,m)]. \tag{3.49}$$

By using (3.15) in (3.49), the noise variance resulting from the K-1 interfering transmitters can be expressed as

$$N_M = \frac{E'\{\epsilon\}}{T} \sum_{\substack{k=1 \\ k \neq i}}^{K} \varepsilon_{k,i} [\mu(2k,n;m)(0)M_\psi + \mu(2k,n;m)(1)M'_\psi + \mu(2k-1,n;m)(0)M_\psi + \mu(2k-1,n;m)(1)M'_\psi] \tag{3.50}$$

If random sequences are used by the interfering transmitters, (3.50) reduces to

$$N_M = 2T_c^{-1}[NM_\psi + C(n,n;m,m)(T_c)M'_\psi]E'\{\epsilon\} \sum_{\substack{k=1 \\ k \neq i}}^{K} \varepsilon_{k,i} . \tag{3.51}$$

Furthermore, if the i-th transmitter also utilizes random sequences, (3.51) reduces to

$$N_M = 2T^{-1}N^2M_\psi E'\{\epsilon\} \sum_{\substack{k=1 \\ k \neq i}}^{K} \varepsilon_{k,i} . \tag{3.52}$$

Suppose the same number of channel paths to the i-th receiver is expected from each transmitter (i.e., $\bar{L}$) and suppose all the paths have the same mean-squared strength (i.e., $E\{g^2(k,i;m)\} = G$). In this case, (3.52) reduces to

$$N_M = 2T^{-1}N^2M_\psi E'\{L\}\bar{L}G^2(K-1) . \tag{3.53}$$

3.5 Average SNR and Average Probability of Bit Error

In this section we evaluate two different approximations to the average probability of bit error of the receiver that we have been considering. The first approximation requires a small amount of computation. It is an appropriate first approximation to the average probability of bit error when the multiple-access interference and the intersymbol interference are the predominant contributions to the noise. The second approximation requires more computation, but it is appropriate for a wider range of system parameters. It is appropriate whether the thermal noise, the intersymbol interference, or the multiple-access interference is the predominant contribution to the noise.

After defining the two approximations to the average probability of bit error, we evaluate the approximations for some examples. We focus on the multiple-access capability and the immunity to intersymbol interference that the direct-sequence spread-spectrum signaling provides. Hence, in the numerical results we neglect the contribution to the noise variance that results from the random path strengths.

3.5.1 An Approximation Which Requires a Small Amount of Computation

In order to define the first approximation, we express the total variance of the noise as the sum of the variances of the uncorrelated noise components. The total noise variance is given by

$$N = N_S + N_I + N_M + N_T \tag{3.54}$$

and can be evaluated using (3.28), (3.33) ,(3.39) , and (3.50). We define the signal-to-noise ratio of the n-th receiver section in terms of the total noise variance as

$$SNR_1^{(n)} = |E'\{y_0(t_m)\}|N^{-1/2}\,, \tag{3.55}$$

where $E'\{y_0(t_m)\}$ is given by (3.25).

When the intersymbol interference and the multiple-access interference are predominant, we approximate the random input to the decision circuit by a Gaussian random variable with mean that is given by (3.25) and variance that is given by (3.54). The corresponding approximation to the average probability of bit error is given by

$$P_{E_1}^{(n)} = Q(SNR_1^{(n)})\,, \tag{3.56}$$

where $Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-u^2/2}du$.

As an example of this approximation, we consider a system that employs random sequences and a rectangular chip waveform. In order to investigate the effects of intersymbol interference and multiple-access interference, we assume that the path strengths are deterministic and that $g^2(k,i;\lambda) = G$ for each λ in the set $\{1,\ldots,L(k,i)\}$ and for each k and i in the set $\{1,\ldots,K\}$. We assume that the expected number of channel paths from each active transmitter to the receiver is given by $\overline{L}$, i.e., we assume $L(k,i) = \overline{L}$ for each k and i in the set $\{1,\ldots,K\}$. Although the results which we plot do not depend appreciably on $\overline{L}$, we are assuming that $\overline{L}\approx 10$. For this example, we choose a linearly decaying path delay density function. Since the expected number of channel paths from the transmitter to the intended receiver is $\overline{L}$, the path delay density function is given by

$$P_D(t) = \frac{2\overline{L}}{\Delta}(1-\frac{t}{\Delta})p_\Delta(t), \tag{3.57}$$

where Δ is the maximum path delay of the the communication channel. We choose Δ=5T, where T is the data bit duration, in the numerical examples. Each transmitter is assumed to transmit the same energy per bit E_b.

The number of channel paths, and hence the number of signal replicas that arrive at the receiver, is a random quantity L. Therefore, the received energy per bit is a random quantity even though we are assuming that the channel path strengths are deterministic. The received energy per bit is given by E_r=GLE$_b$, where E_b is the transmitted energy per bit. We denote the expected received energy per bit by $\overline{E}_r$.

In Figure 3.3, the approximation given by (3.56) is shown for subsequences of length N=31 and several different values of K. The average probability of bit error for one channel of the quadriphase receiver is plotted as a function of the expected received-energy-per-bit-to-noise-density ratio, i.e., as a function of $\overline{E}_r/N_0$.

Notice that as the transmitted power, and hence the parameter $\overline{E}_r$, is increased beyond a certain level, the performance of the system does not continue to improve. For the case of a single active transmitter, this is because of the intersymbol interference, which increases as the transmitted power

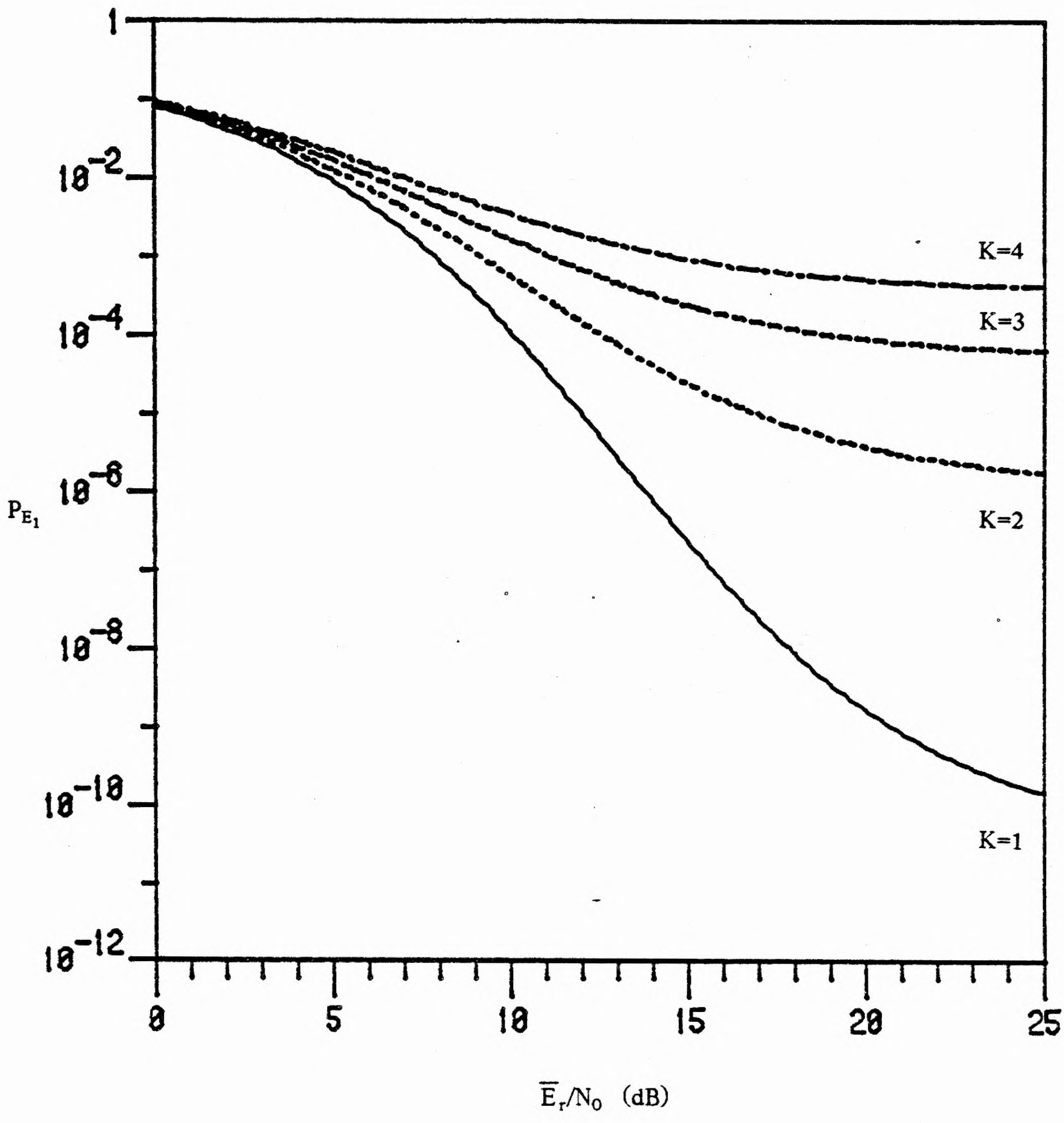

Figure 3.3. Average probability of bit error vs. $\overline{E}_r/N_0$ ($N=31$, $\Delta=5T$, rectangular chip waveform, random sequences).

increases. For the case of multiple active transmitters with equal power levels, this is because all transmitters are increasing the transmitted power together. Hence, the multiple-access interference and the intersymbol interference both increase as the transmitted power increases throughout the system.

In Figure 3.4, we plot the average probability of bit error versus the number of active transmitters K for a fixed value of $\overline{E}_r/N_0$ which is equal to 10dB. The plots are given for various lengths of the signature subsequences and for a rectangular chip waveform. The maximum path delay is again given by $\Delta=5T$, and the path delay density function is given by (3.57). Again, we plot the results for one channel of the quadriphase receiver.

3.5.2 The Special Case of Very Long Signature Subsequences

As the signature subsequences become long, the multiple-access interference and the intersymbol interference become small compared to the thermal noise at the receiver. In the case of very long signature subsequences, we can neglect the multiple-access interference and the intersymbol interference when we compare them with the thermal noise at the receiver. Assuming that the number of signal paths from the transmitter to the intended receiver is known to be L, the signal-to-noise ratio for this special case (again assuming deterministic path strengths) can be found from modifications of (3.26) and (3.40) and is given by

$$SNR_s(L)=\frac{TGL}{\sqrt{N_T}}=\sqrt{\frac{2E_bGL}{N_0}}=\sqrt{\frac{2E_r}{N_0}}, \tag{3.58}$$

where E_b and E_r are the transmitted energy per bit and the received energy per bit, respectively.

Notice once again that the number L of signal paths from the transmitter to the intended receiver is a random quantity so the received energy per bit is a random quantity even though we are assuming that the channel path strengths are deterministic. However, we can express the signal-to-noise ratio in terms of a deterministic quantity which is directly related to the transmitted energy per bit. In terms of the expected received energy per bit $\overline{E}_r$ the signal-to-noise ratio is given by

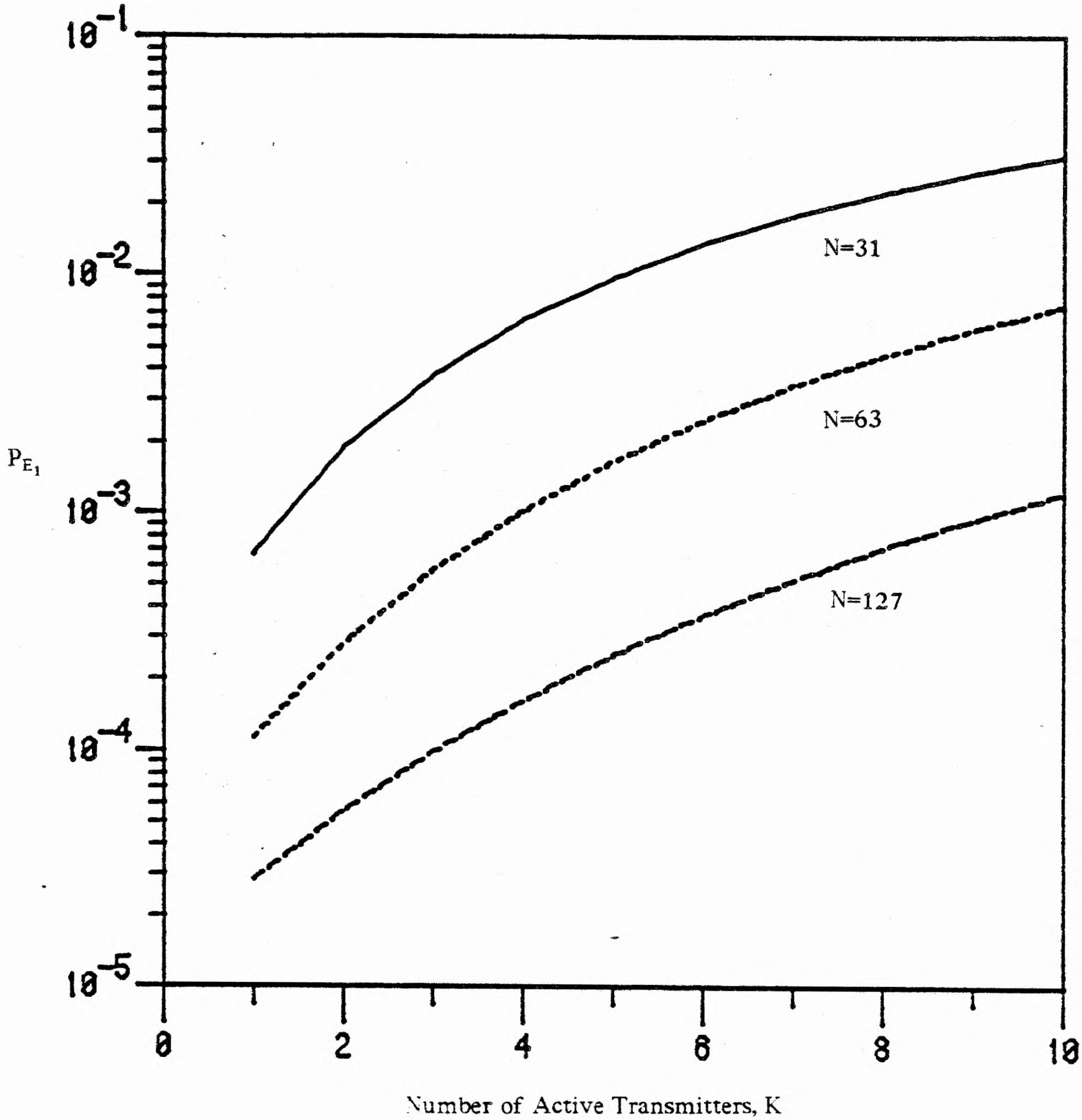

Figure 3.4. Average probability of bit error vs. number K of active transmitters ($\overline{E}_r/N_0$=10dB , Δ=5T, rectangular chip waveform, random sequences).

$$\mathrm{SNR}_s(L)=\sqrt{\frac{2\overline{E}_r L}{N_0\overline{L}}}. \tag{3.59}$$

For a fixed number of signal paths L, the average probability of bit error is given by

$$P_{E_s}(L)=Q(\mathrm{SNR}_s(L)). \tag{3.60}$$

The overall average probability of bit error is given by

$$E'\{P_{E_s}(L)\} = E'\{Q(\mathrm{SNR}_s(L))\}. \tag{3.61}$$

The approximation of (3.61) becomes better and better as the length of the signature subsequences grows. In fact, the approximation approaches the exact value of the average probability of bit error in the limiting case since the thermal noise is indeed Gaussian. Furthermore, the expression in (3.61) can be conveniently evaluated since in our model L is a Poisson random variable with mean $\overline{L}$.

The importance of (3.61) is that it demonstrates that the simple approximation of equation (3.56) is not accurate for some system parameters. If the thermal noise is the predominant contribution to the total noise, the simple approximation of (3.56) is significantly different from the approximation of (3.61) even though the approximation of (3.61) becomes closer and closer to the exact value of the average probability of bit error as the length of the signature subsequences grows. The reason for this discrepancy is that the Gaussian approximation to the random output of the receiver is very accurate in this special case when conditioned on the random variable L, but very poor when the random output of the receiver is not conditioned on L. This suggests the approximation of the next section in which a Gaussian approximation is applied to the random output of the receiver only after the random output is conditioned on some key random variables.

3.5.3 A More Accurate Approximation

In this section we develop a more accurate approximation to the average probability of bit error that is applicable to a wider range of system conditions. We begin by defining a random variable M_i, where the subscript i is an index for the appropriate receiver, as

$$M_i = \sum_{\substack{k=1 \\ k\neq i}}^{K} L(k,i). \tag{3.62}$$

For a fixed i in the set {1,...K}, the random variables L(k,i) for k in the set {1,...K} form a set of mutually independent Poisson random variables. Therefore, the random variable M_i is also a Poisson random variable with a mean that is given by

$$\overline{M}_i = \sum_{\substack{k=1 \\ k\neq i}}^{K} \overline{L}(k,i), \tag{3.63}$$

where $\overline{L}(k,i)$ denotes the mean of the random variable L(k,i). In order to simplify notation, we define $M=M_i$. In the following we define a signal-to-noise ratio that is a function of the random variables L and M and make an approximation to the average probability of bit error which involves this signal-to-noise ratio. We then average with respect to the random variables L and M to obtain an overall approximation.

We first find the contribution $N_I(L)$ to the total noise variance that results from intersymbol interference when given the number of paths L from the transmitter to the intended receiver. The parameter $N_I(L)$ can be found from modifications of (3.33) and (3.35) The parameter $N_I(L)$ is again given by (3.33), but the function D(x) is now given for $|t| \geq T_c$ by

$$D(x) = \frac{1}{2} G^2 \frac{L(L-1)}{\overline{L}^2} P_D(x) * P_D(-x), \tag{3.64}$$

by $N_T(L) = LGN_0T$. Finally, the desired signal component for a given value of L is found from a modification of (3.26) to be $S_D(L)=b_m^{(n)}TLG$.

We are now in a position to define a signal-to-noise ratio for given values of L and M as

$$SNR_2(L,M) = \frac{S_D(L)}{\sqrt{N_I(L)+N_M(L,M)+N_T(L)+N_s(L)}}, \tag{3.65}$$

where for the numerical results which we consider $N_s(L)=0$. Our approximation to the average probability of bit error for known values of L and M is given by

$$P_{E_2}(L,M) = Q(SNR_2(L,M)), \tag{3.66}$$

and the overall approximation to the average probability of bit error is given by

$$P_{E_2} = E\{P_{E_2}(L,M)\} = E\{Q(SNR_2(L,M)\}. \tag{3.67}$$

The accuracy of an approximation of a similar form, when applied to a system employing a simple correlation receiver and an AWGN channel, is studied in [4].

In Figure 3.5, we plot this second approximation to the average probability of bit error versus $\overline{E}_r/N_0$ for the same system parameters as in Figure 3.3. In Figure 3.6, the parameters are again the same except for the length of the signature subsequences, which is now N=127. In Figure 3.7, we plot the average probability of bit error which is given by this second approximation as a function of the number of active transmitters. The system parameters are identical to those of Figure 3.4.

So far the numerical results that we have given for the average probability of bit error have been for a fixed number of active transmitters. In a packet radio system the number of active radios is a random variable κ. It may be of interest then to evaluate the average probability of bit error with respect to the number of active transmitters as well as with respect to the other random quantities in

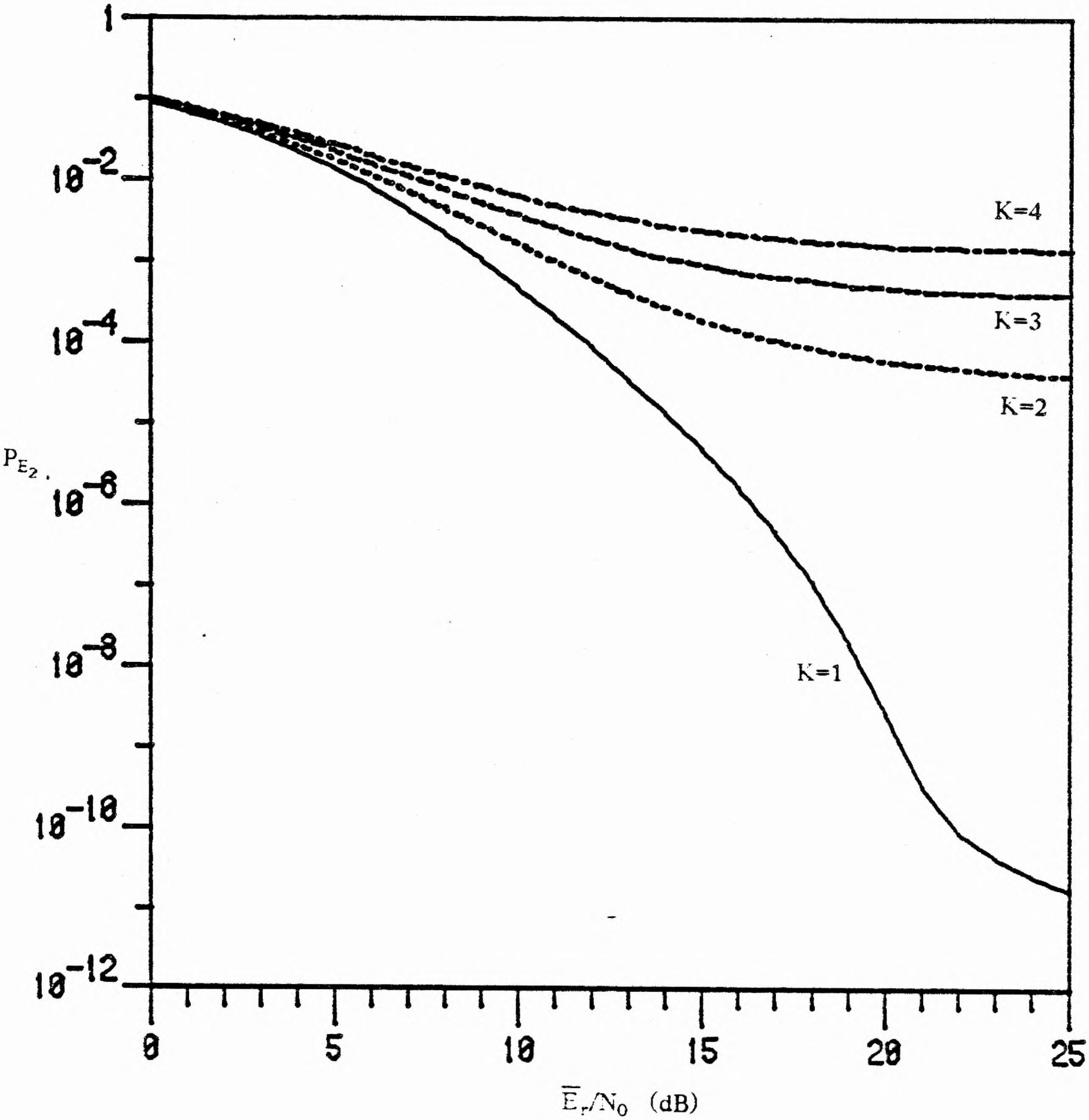

Figure 3.5. Average probability of bit error vs. $\overline{E}_r/N_0$ (N=31, Δ=5T, rectangular chip waveform, random sequences).

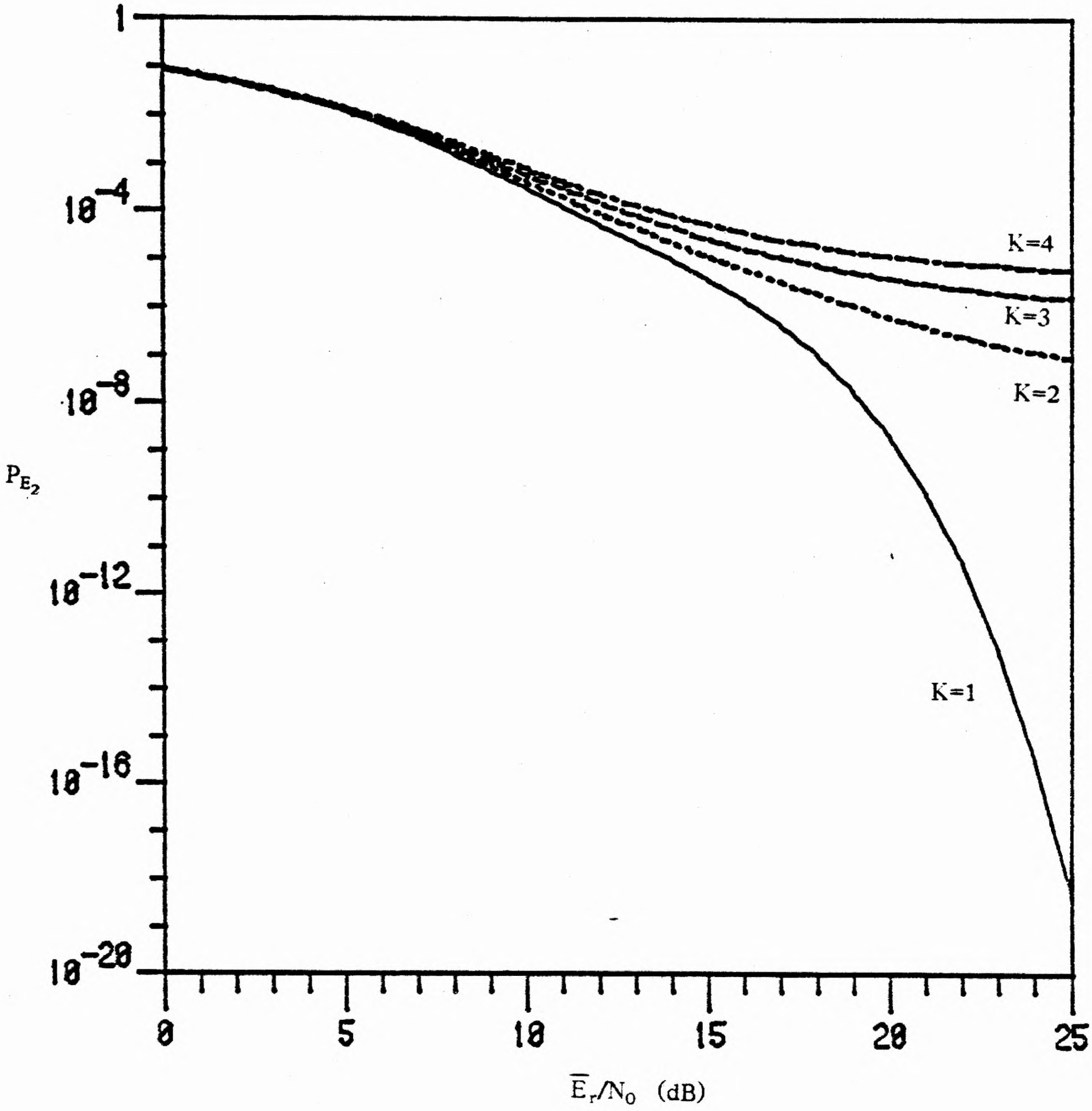

Figure 3.6. Average probability of bit error vs. $\overline{E}_r/N_0$ (N=127, Δ=5T, rectangular chip waveform, random sequences).

except for the length of the signature subsequences, which is now N=127. In Figure 3.7, we plot the average probability of bit error which is given by this second approximation as a function of the number of active transmitters. The system parameters are identical to those of Figure 3.4.

So far the numerical results that we have given for the average probability of bit error have been for a fixed number of active transmitters. In a packet radio system the number of active radios is a random variable κ. It may be of interest then to evaluate the average probability of bit error with respect to the number of active transmitters as well as with respect to the other random quantities in the system. The results for a fixed number of active transmitters that we have plotted can be extended to the case of a random number of active transmitters by using (2.2).

3.6 Conclusions

It is possible to draw several important conclusions from the numerical results of the previous section. First of all, for a receiver of information through a specular multipath channel, the approximations show that substantial gains can be achieved if the parameters of the channel can be ascertained. In fact, the complex receiver that we have described can achieve a performance comparable to the performance of a simple correlation receiver at a much smaller (by a factor of $\overline{L}$) transmitted power level. The plots for the simple correlation receiver look very much like those which we have plotted with the expected received energy per bit $\overline{E}_r$ scaled by a factor of $\overline{L}$ [11].

It is warranted then to attempt to build a more complex receiver, which by ascertaining the channel parameters is able to combine the information inherent in the several transmitted signal replicas that are present at the input to the receiver. A practical receiver will not be able to achieve a perfect knowledge of the communication channel. However, since the possible gains are very large, we expect the performance to improve even with an imperfect knowledge of the channel parameters.

We can also see by comparing the results of [3] with those of this chapter that the multiple-access capability of the system is about the same as the multiple-access capability that we would expect from a system consisting of a simple correlation receiver and an AWGN channel. Although the multiple

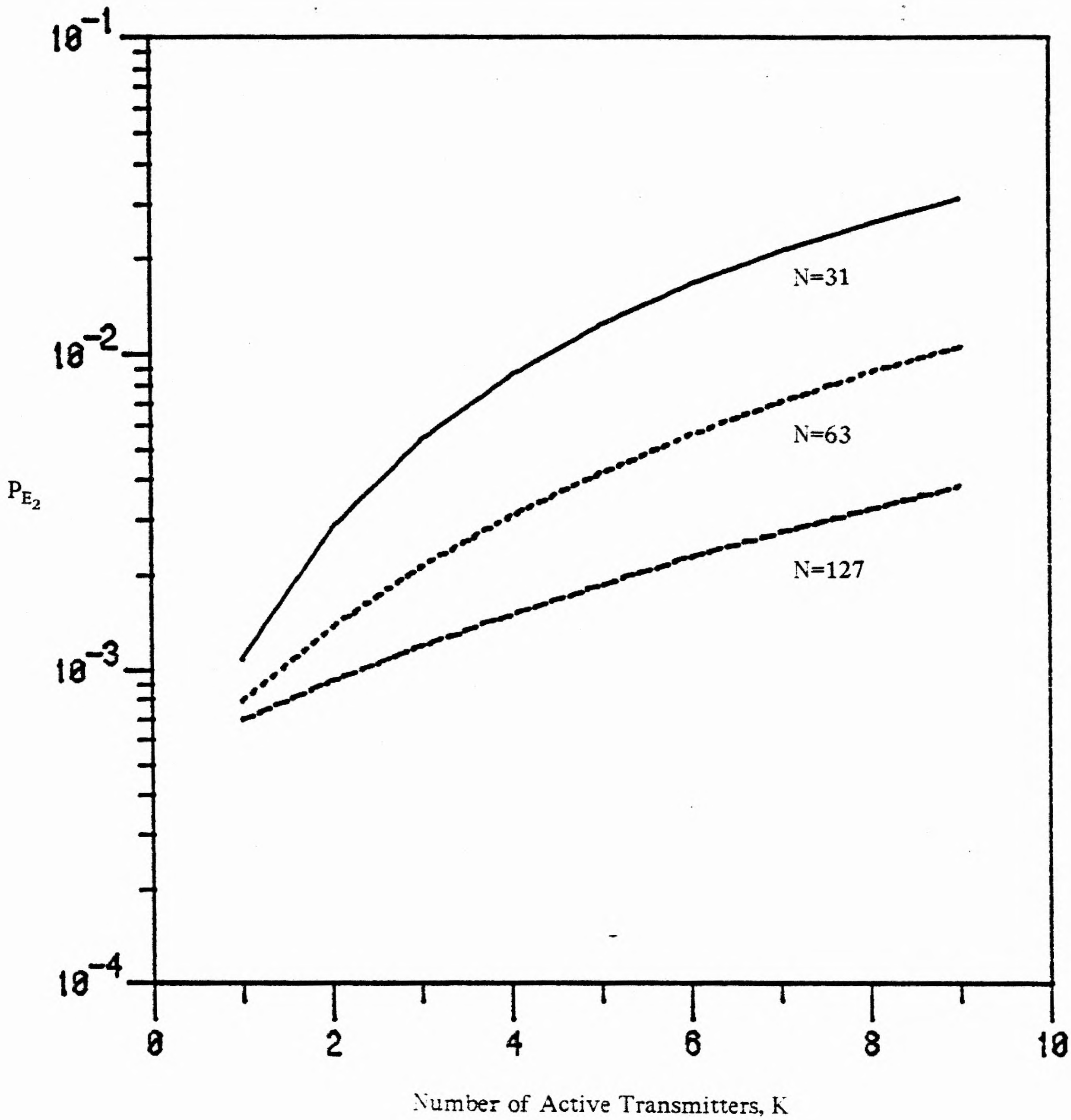

Figure 3.7. Average probability of bit error vs. number K of active transmitters ($\overline{E}_r/N_0$=10dB , Δ=5T, rectangular chip waveform, random sequences).

signal paths from an interfering transmitter to a receiver increase the multiple-access interference, the added interfence is compensated by the increased received signal power that results from the multiple signal paths from the desired transmitter. If the simple correlation receiver is used in the system, however, or if the receiver is unable to ascertain the channel parameters, the multiple-access capability of a system with a specular multipath channel is greatly degraded.

We have identified key parameters of the signature subsequences that influence the multiple-access capability of the system and the immunity of the system to intersymbol interference. In fact, a comparison of the results of [3] and the results of Section 3.4.3 reveals that the key parameters of the signature subsequences that influence the multiple-access capability of the system are similar to the key parameters identified in [3] for the AWGN channel.

The average probability of bit error of the receiver we have considered can be evaluated with more and more accuracy as the amount of computation that is performed increases. We have described two approximations to the average probability of bit error. The second approximation is better for some system conditions, but requires more computation. It is possible to evaluate the characteristic function of the random output of the receiver in order to obtain an approximation to the average probability of bit error as is done in [5] and [11], but because of the dependence of the various terms in the expression for the random output of the receiver, this requires very large amounts of computation. The computational simplifications that are possible in [11] because of the independence of various terms in the expression for the random output of the receiver do not occur as conveniently for this receiver.

CHAPTER 4

SPREAD-SPECTRUM SIGNALING THROUGH THE AWGN CHANNEL

4.1 Introduction

In Chapter 3, we considered a complex communication channel and obtained the signal-to-noise ratio of a receiver that had been designed for that particular channel. Because of the complexity of the system, the exact value of the average probability of bit error was not obtained. In this chapter a simpler communication channel and a simpler receiver are considered, and the exact value of the average probability of bit error of this simpler receiver is determined.

In this chapter we are concerned with direct-sequence spread-spectrum multiple-access (DS/SSMA) communications, an additive white Gaussian noise (AWGN) channel, and a coherent correlation receiver. We assume that all the signature sequences are randomly generated sequences. An expression is given for the output of a correlation receiver in terms of a set of mutually independent random variables. An expression is also given for the probability density function of each of the random variables in the set. These expressions are then used to obtain arbitrarily tight upper and lower bounds on the average probability of bit error.

We are also interested in the performance of the receiver when it is used as a component in a packet radio system. In a packet radio system an additional structure is imposed on the transmitted data. For example, the transmitted data might be organized into packets of consecutive data bits. An error-correcting code may be employed so that a number of data bit errors in each packet can be tolerated. If more data bit errors occur in the packet than the error-correcting code can correct, the packet is received in error.

The average probability of bit error may not be the performance measure of the receiver which is of most importance in the examination of a packet radio network. An important performance measure in a packet radio system is the packet error probability. The packet error probability is the probability that a packet of digital data must be transmitted again in order to convey all of the information in the packet correctly.

The packet error probability can be difficult to evaluate analytically. When direct-sequence spread-spectrum signaling is employed, the probability that an error occurs in detecting a given data bit is strongly dependent on whether contiguous bits are detected incorrectly. If the data bits are grouped sequentially into packets, it is difficult to evaluate the performance of the system. The expression for the output of the correlation receiver which we obtain in this chapter is given in a form which is particularly useful for applications to packet radio systems.

4.2 System Model

The system model which we consider in this chapter is similar to the models which have been defined in [2] and [3]. The channel model is a specialization of the general specular multipath channel which is defined in Chapter 2. In this chapter we consider the model in which there is just one path from each active transmitter to the receiver. Furthermore, the signals arriving at the receiver are assumed to be unattenuated. In the terminology of Chapter 2, $L(k,i) = 1$ for all k and i. Also $g(k,i;\lambda)=1$ for all k, i, and λ. The transmitter and receiver models are those of Chapter 3 specialized to the case of BPSK signaling in which the signature sequences are randomly generated. (Although the following analysis is restricted to the case of BPSK signaling, we state corresponding results for quaternary systems in Appendix A.) Since the more specialized AWGN channel is being considered in this chapter, the receiver reduces to a simple correlation receiver.

The model that we are now considering is shown in Figure 4.1. The received signal in this asynchronous binary DS/SSMA system is the sum of K spread-spectrum signals $s_k(t-\tau_k)$, $1 \leqslant k \leqslant K$, plus an AWGN process $n(t)$ which has two-sided spectral density $N_0/2$. The spread-spectrum signal $s_k(t-\tau_k)$ is given by

$$s_k(t-\tau_k)=\sqrt{2P}b_k(t-\tau_k)a_k(t-\tau_k)\cos(\omega_c t+\phi_k), \tag{4.1}$$

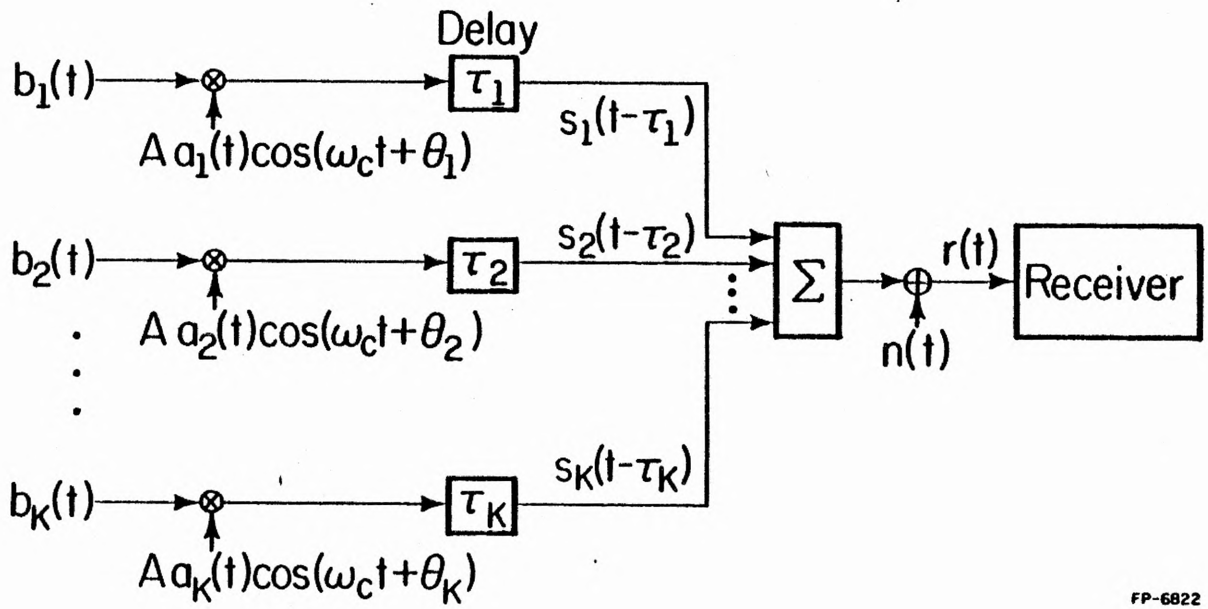

Figure 4.1. DS/SSMA system model.

where $b_k(t)$ is the data signal, $a_k(t)$ is the spectral-spreading signal, τ_k is the time delay parameter which accounts for propagation delay and the lack of synchronism between the transmitters, ϕ_k is the phase angle of the k-th carrier, and P is the power of the transmitted signal. Although equal power levels have been assumed for all transmitters, the results are easily modified to consider unequal power levels. Notice in Figure 4.1, $A=\sqrt{2P}$, and $\phi_k=\theta_k-\omega_c\tau_k$.

If we define the unit pulse function $p_T(t)$ by $p_T(t)=1$ for $0 \leqslant t < T$, and $p_T(t)=0$ otherwise, the k-th data signal can be expressed as

$$b_k(t) = \sum_{j=-\infty}^{\infty} b_j^{(k)} p_T(t-jT). \tag{4.2}$$

The sequence $(b_j^{(k)})$ is the binary data sequence of the k-th transmitter ($b_j^{(k)} \in \{-1,1\}$ for each j).

It is important to define carefully the spectral-spreading signal $a_k(t)$. We again define a chip waveform $\psi(t)$ of arbitrary shape which is time limited to the interval $[0,T_c)$ and is normalized such that $\int_0^{T_c} \psi^2(t)=T_c$. The spectral-spreading signal of the k-th transmitter may now be expressed as

$$a_k(t) = \sum_{j=-\infty}^{\infty} a_j^{(k)} \psi(t-jT_c), \tag{4.3}$$

where $(a_j^{(k)})$ is a periodic binary sequence of elements from the set $\{-1,1\}$. We assume that each bit is encoded with N chips, i.e. $T=NT_c$, and that the signature sequence $(a_j^{(k)})$ has period N. Notice that if the chip waveform $\psi(t)$ is the rectangular pulse function $p_{T_c}(t)$, the DS/SSMA system has the binary phase-shift-keyed signaling format.

The signature sequences being used in the system are deterministic. Each transmitter and receiver pair has been designed to encode and then decode data using a particular sequence. We assume in our model that the sequences have been randomly generated, however. Instead of carefully choosing a signature sequence for each transmitter and receiver pair, a signature sequence has been chosen at random for the pair. We assume the sequence for each pair has been chosen from the set of all possible sequences and that each sequence in the set of all possible sequences has an equal probability of being chosen. Furthermore, we assume that the choice of a sequence for each transmitter and receiver pair is made independently of the choices made for all other pairs. This means that in our model it is possible, but very improbable, that more than one transmitter will use the same sequence. The average probability of bit error which we obtain is an average with respect to all the possible combinations of signature sequences which might be used in the system.

It is useful at this point to list some of the random variables which model the communication system and to describe their distributions. First, there are K transmitter and receiver pairs so in what follows $1 \leqslant k \leqslant K$. The k-th data sequence is modeled as a sequence of independent and identically distributed random variables $(b_j^{(k)})$ such that $\Pr\{b_j^{(k)} = +1\} = \Pr\{b_j^{(k)} = -1\} = 1/2$. The k-th signature sequence is periodic with period N. One period of the sequence is modeled by a random vector $[a_0^{(k)}, a_1^{(k)}, \ldots, a_{N-1}^{(k)}]$ of length N. The components of this vector $a_j^{(k)}$ for $0 \leqslant k \leqslant N-1$ are a set of independent, identically distributed random variables such that $\Pr\{a_j^{(k)} = +1\} = \Pr\{a_j^{(k)} = -1\} = 1/2$. Because of the symmetry of the system, we need only consider the receiver which is listening to the first transmitter. Also, since only relative delays and phases are important, we set $\tau_1 = \phi_1 = 0$. The properties of an SSMA system and the stationarity of the noise n(t) permit us to consider only time delays modulo T and phase angles modulo 2π, rather than the absolute values of these parameters. Hence, for $k \neq 1$, we model the delay τ_k as a random variable which is uniformly distributed on [0,T) and the phase ϕ_k as a random variable which is uniformly distributed on $[0,2\pi)$. Finally, we assume that the collection of all the random variables mentioned in this paragraph forms a set of mutually independent random variables.

4.3 System Analysis

The output statistic of the receiver for the case of deterministic sequences and coherent detection of BPSK signaling has been shown in [3] to be given by

$$Z_{out}^{(1)} = \eta + T\sqrt{P/2}\left[b_1^{(1)} + \sum_{k=2}^{K} I_{k,1}(\underline{b}_k,\tau_k,\phi_k)\right], \tag{4.4}$$

where

$$I_{k,1}(\underline{b}_k,\tau,\phi) = T^{-1}[B_{k,1}(\underline{b}_k,\tau)]\cos\phi \tag{4.5}$$

and

$$B_{k,1}(\underline{b}_k,\tau) = b_1^{(k)}R_{k,1}(\tau) + b_2^{(k)}\hat{R}_{k,1}(\tau). \tag{4.6}$$

The random variable η is Gaussian with mean zero and variance equal to $N_0T/4$, where $N_0/2$ is the two-sided density of white Gaussian noise. P denotes the power of each transmitter's signal, and T denotes the data bit duration. The vector $\underline{b}_k=(b_1^{(k)},b_2^{(k)})$ represents a pair of consecutive data bits of the k-th signal. The functions $R_{k,m}(\tau)$ and $\hat{R}_{k,m}(\tau)$ are the continuous-time partial crosscorrelation functions of the k-th and the m-th spectral-spreading waveforms which have been defined in [2] and [3] to be given by

$$R_{k,i}(\tau) = \int_0^{\tau} a_k(t-\tau)a_i(t)dt \tag{4.7}$$

and

$$\hat{R}_{k,i}(\tau) = \int_{\tau}^{T} a_k(t-\tau)a_i(t)dt \tag{4.8}$$

for $0 \leqslant \tau \leqslant T$.

The continuous-time partial crosscorrelation functions of the k-th and the m-th signature sequences, $R_{k,m}(\cdot)$ and $\hat{R}_{k,m}(\cdot)$, can be expressed in terms of the discrete aperiodic crosscorrelation function $C_{k,m}(\cdot)$ and the continuous-time partial autocorrelation functions of the chip waveform $R_\psi(\cdot)$ and $\hat{R}_\psi(\cdot)$. The discrete aperiodic crosscorrelation function $C_{k,m}(\cdot)$ involves only the k-th and m-th signature sequences and is given by

$$C_{k,m}(\lambda) = \begin{cases} \sum_{j=0}^{N-1-\lambda} a_j^{(k)} a_{j+\lambda}^{(m)}, & 0 \leqslant \lambda \leqslant N-1 \\ \sum_{j=0}^{N-1+\lambda} a_{j-\lambda}^{(k)} a_j^{(m)}, & 1-N \leqslant \lambda < 0, \\ 0, & |\lambda| \geqslant N \end{cases} \tag{4.9}$$

where $a_i^{(k)}$ and $a_j^{(m)}$ are elements of the sequences $(a_i^{(k)})$ and $(a_j^{(m)})$. The continuous-time partial autocorrelation functions of the chip waveform have been defined in (3.12) and (3.13).

The continuous-time partial crosscorrelation functions of the k-th and m-th signature sequences, $R_{k,m}(\cdot)$ and $\hat{R}_{k,m}(\cdot)$, depend on the k-th and m-th signature sequences only through the function $C_{k,m}(\cdot)$. The functions depend on the chip waveform only through the functions $R_\psi(\cdot)$ and $\hat{R}_\psi(\cdot)$. The dependence is governed for $0 \leqslant \tau \leqslant T$ by the equations

$$R_{k,m}(\tau) = C_{k,m}(\gamma-N)\hat{R}_\psi(\tau-\gamma T_c) + C_{k,m}(\gamma+1-N)R_\psi(\tau-\gamma T_c) \tag{4.10}$$

and

$$\hat{R}_{k,m}(\tau) = C_{k,m}(\gamma)\hat{R}_\psi(\tau-\gamma T_c) + C_{k,m}(\gamma+1)R_\psi(\tau-\gamma T_c), \tag{4.11}$$

where $\gamma = \lfloor \tau/T_c \rfloor$.

We now return to the examination of (4.4). From (4.6), (4.10), and (4.11) we find

$$B_{k,1}(\underline{b}_k,\tau_k) = V_{k,1}(\gamma_k)\hat{R}_\psi(S_k) + U_{k,1}(\gamma_k)R_\psi(S_k), \tag{4.12}$$

where

$$V_{k,i}(\gamma_k) = b_1^{(k)}C_{k,i}(\gamma_k-N) + b_2^{(k)}C_{k,i}(\gamma_k), \tag{4.13}$$

and

$$U_{k,i}(\gamma_k) = b_1^{(k)}C_{k,i}(\gamma_k+1-N) + b_2^{(k)}C_{k,i}(\gamma_k+1), \tag{4.14}$$

$S_k=\tau_k-\gamma_k T_c$, and $\gamma_k=\lfloor \tau_k/T_c \rfloor$. Since τ_k is a random variable which is uniformly distributed on $[0,T)$, S_k is a random variable which is uniformly distributed on $[0,T_c)$, and γ_k is a random variable which is uniformly distributed on the set $\{0,\ldots,N-1\}$.

At this point in order to simplify notation we define $(y_i)=(a_i^{(1)})$, $(x_i)=(a_i^{(k)})$, $b_1=b_1^{(k)}$, $b_2=b_2^{(k)}$, $\tau=\tau_k$, $\gamma=\gamma_k$, and $S=S_k$. When we need to consider data sequences, signature sequences, and delays from a number of users, we can restore the appropriate indices.

If we use (4.9), (4.13), and (4.14) to expand (4.12), we obtain

$$B_{k,1}(\underline{b}_k,\tau) = \left[\sum_{j=0}^{\gamma-1} b_1 x_{j-\gamma+N} y_j + \sum_{j=\gamma}^{N-1} b_2 x_{j-\gamma} y_j\right] \hat{R}_\psi(S) \tag{4.15}$$
$$+ \left[\sum_{j=0}^{\gamma} b_1 x_{j-\gamma-1+N} y_j + \sum_{j=\gamma+1}^{N-1} b_2 x_{j-\gamma-1} y_j\right] R_\psi(S).$$

Equation (4.15) may be further expanded to obtain

$$B_{k,1}(\underline{b}_k,\tau) = \left[\sum_{j=0}^{\gamma-1} b_1 x_{j-\gamma+N} y_j + \sum_{j=\gamma}^{N-2} b_2 x_{j-\gamma} y_j + b_2 x_{N-\gamma-1} y_{N-1}\right] \hat{R}_\psi(S) \tag{4.16}$$
$$+ \left[b_1 x_{N-\gamma-1} y_0 + \sum_{j=0}^{\gamma-1} b_1 x_{j-\gamma+N} y_{j+1} + \sum_{j=\gamma}^{N-2} b_2 x_{j-\gamma} y_{j+1}\right] R_\psi(S).$$

Finally, the terms of (4.16) can be rearranged to give

$$B_{k,1}(\underline{b}_k,\tau) = b_1 \sum_{j=0}^{\gamma-1} x_{j-\gamma+N} \left(y_j\hat{R}_\psi(S) + y_{j+1}R_\psi(S)\right) \tag{4.17}$$

$$+ b_2 \sum_{j=\gamma}^{N-2} x_{j-\gamma} \left(y_j\hat{R}_\psi(S) + y_{j+1}R_\psi(S)\right)$$

$$+ b_2 x_{N-\gamma-1} y_{N-1} \hat{R}_\psi(S) + b_1 x_{N-\gamma-1} y_0 R_\psi(S).$$

We are interested in an average performance with respect to all the possible combinations of signature sequences of length N which might be used in the system. Since there are 2^N possible sequences for each of K transmitters, there are 2^{KN} possible combinations to consider. This number can be too large to perform practical computations even when the sequence length N is small, e.g., N=31. For this reason it is necessary to carefully manipulate (4.17) in order to obtain an expression for the output statistic of the receiver in a form which is useful for practical computations.

With the motivation of reducing complexity, we consider (4.17) conditioned on the signature sequence of the first receiver (y_j) and the random variable γ, which is uniformly distributed on the set $\{0,\dots,N-1\}$. It is very important to condition the output statistic on the signature sequence of the first receiver (or, as we will later demonstrate, just the one parameter $C_{1,1}(1)$ of that sequence) before proceeding with the analysis. Without this conditioning, the random variables which model the multiple-access interference from the multiple transmitters are not a set of independent random variables, and our expression for the output statistic loses its utility. We assume $\gamma=\hat{\gamma}$ and $(y_j)=(\hat{y}_j)$. In order to simplify (4.17), we define a set of N+1 random variables Z_j for $0 \leqslant j \leqslant N$ by

$$Z_j = \begin{cases} b_1 x_{j-\hat{\gamma}+N} \hat{y}_j\,, & j=0,\dots,\hat{\gamma}-1 \\ b_2 x_{j-\hat{\gamma}} \hat{y}_j\,, & j=\hat{\gamma},\dots,N-2 \\ \hat{y}_{N-1} b_2 x_{N-\hat{\gamma}-1}\,, & j=N-1 \\ \hat{y}_0 b_1 x_{N-\hat{\gamma}-1}\,, & j=N \end{cases} \tag{4.18}$$

For any $\hat{\gamma}$ in the set $\{0,\dots,N-1\}$, the random variables Z_j for $0 \leqslant j \leqslant N$ are a set of N+1 mutually independent and identically distributed random variables such that $\Pr\{Z_j = +1\} = \Pr\{Z_j = -1\} = 1/2$. One might first toss a fair coin to determine $x_{N-\hat{\gamma}-1}$. A second toss would determine b_1 and therefore

$Z_N=\hat{y}_0 b_1 x_{N-\hat{\gamma}-1}$. A third toss would determine b_2 and therefore $Z_{N-1}=\hat{y}_{N-1} b_2 x_{N-\hat{\gamma}-1}$. The remaining $N-1$ tosses would determine $x_{j-\hat{\gamma}+N}$ and hence $Z_j=b_1 x_{j-\hat{\gamma}+N}\hat{y}_j$ for $j=0,\ldots,\hat{\gamma}-1$, and $x_{j-\hat{\gamma}}$ and hence $Z_j=b_2 x_{j-\hat{\gamma}}\hat{y}_j$ for $j=\hat{\gamma},\ldots,N-2$.

Using our definition of the random variables Z_j for $0\leqslant j\leqslant N$ and the fact that $\hat{y}_j^2=1$ for each j, (4.17) may be simplified to

$$B_{k,1}(\underline{b_k},\tau) = \sum_{j=0}^{N-2} Z_j(\hat{R}_\psi(S) + \hat{y}_j\hat{y}_{j+1}R_\psi(S)) + Z_{N-1}\hat{R}_\psi(S) + Z_N R_\psi(S), \tag{4.19}$$

where the random variables Z_j for $0\leqslant j\leqslant N$ are a set of N+1 mutually independent and identically distributed random variables such that $\Pr\{Z_j = +1\} = \Pr\{Z_j = -1\} = 1/2$.

For notational convenience we define

$$f(s) = \hat{R}_\psi(s) + R_\psi(s) \tag{4.20}$$

and

$$g(s) = \hat{R}_\psi(s) - R_\psi(s). \tag{4.21}$$

We also define the sets

$$A = \{0,\ldots,N-2\} \bigcap \{i : \hat{y}_i\hat{y}_{i+1} = 1\} \tag{4.22}$$

and

$$B = \{0,\ldots,N-2\} \bigcap \{i : \hat{y}_i\hat{y}_{i+1} = -1\} \tag{4.23}$$

in order to strategically split the sum of (4.19). The structure of (4.19) can now be exploited by writing it as

$$B_{k,1}(\underline{b}_k,\tau) = \sum_{j\in A} Z_j\, f(S) + \sum_{j\in B} Z_j\, g(S) + Z_{N-1}\hat{R}_\psi(S) + Z_N R_\psi(S). \quad \textbf{(4.24)}$$

If we restore the index k which has been implicit in the preceding paragraphs, we obtain

$$B_{k,1}(\underline{b}_k,\tau_k) = X_k f(S_k) + Y_k g(S_k) + P_k\hat{R}_\psi(S_k) + Q_k R_\psi(S_k), \quad (4.25)$$

where

$$X_k = \sum_{j\in A} Z_j\,, \quad (4.26)$$

$$Y_k = \sum_{j\in B} Z_j\,, \quad (4.27)$$

$P_k = Z_{N-1}$, and $Q_k = Z_N$.

The random variable X_k is a function of the elements of the set $\{Z_j : j\in A\}$, and the random variable Y_k is a function of the elements of the set $\{Z_j : j\in B\}$. Furthermore, the sets A, B, {N-1}, and {N} are mutually disjoint. Since the random variables Z_j for $0\leqslant j\leqslant N$ are mutually independent, the random variables X_k, Y_k, P_k, and Q_k are mutually independent.

By using (4.4), (4.5), and (4.25), we now may express the output statistic of the receiver in the simplified form

$$Z_{out}^{(1)} = \eta + b_1^{(1)}T\sqrt{P/2} + \sqrt{P/2}\sum_{k=2}^{K} W_k, \quad (4.28)$$

where

$$W_k = [P_k\hat{R}_\psi(S_k)+Q_k R_\psi(S_k)+X_k f(S_k)+Y_k g(S_k)]\cos\phi_k. \quad (4.29)$$

Furthermore, the random variables W_k for $2\leqslant k\leqslant K$ are a set of mutually independent random variables. This follows from the fact that each of these random variables is a function of the elements in a subset of a set of mutually independent random variables and from the fact that the K-1 subsets corresponding to the random variables W_k for $2 \leqslant k \leqslant K$ are mutually disjoint.

The probability density functions of P_k and Q_k for $2 \leqslant k \leqslant K$ are already known, and the probability density functions of X_k and Y_k can be determined by elementary combinatorial arguments. If we denote the cardinality of the set A by $|A|$ and the cardinality of the set B by $|B|$, the probability density function of the discrete random variable X_k is given by

$$p_{X_k}(j) = C\left(|A|, \frac{j+|A|}{2} \right) 2^{-|A|}, \quad j = -|A|, -|A|+2, \ldots, |A|-2, |A|, \tag{4.30}$$

and the probability density function of the discrete random variable Y_k is given by

$$p_{Y_k}(j) = C\left(|B|, \frac{j+|B|}{2} \right) 2^{-|B|}, \quad j = -|B|, -|B|+2, \ldots, |B|-2, |B|, \tag{4.31}$$

where in the above equations the function $C(n,k)$ represents the binomial coefficient $\binom{n}{k}$. The probability densities $p_{X_k}(i)$ and $p_{Y_k}(j)$ are nonzero only for the discrete values specifically mentioned in (4.30) and (4.31).

It is helpful at this point to examine our progress in reducing the complexity of (4.4). The computational problem involved in considering all the 2^{KN} possible combinations of signature sequences has already been mentioned. If we examine (4.29), we see that conditioned on the signature sequence of the first receiver, the random variable W_k depends on the k-th signature sequence only through the mutually independent random variables P_k, Q_k, X_k, and Y_k. There are two possible values for the random variable P_k and two possible values for the random variable Q_k. There are $|A|+1$ possible values for the random variable X_k, and $|B|+1$ possible values for the random variable Y_k. This means, instead of considering the 2^N possibilities for the k-th signature sequence, we need only consider $4(|A|+1)(|B|+1)$ possible combinations of the values of these four random variables. Since

$|A|+|B|=N-1$, the number $4(|A|+1)(|B|+1)$ is significantly smaller than 2^N for any practical number N of chips per data bit. In fact, this product is less than or equal to $(N+1)^2$. Furthermore, we have established that the random variables W_k for $2 \leqslant k \leqslant K$ are a set of mutually independent and identically distributed random variables. If we consider the $4(|A|+1)(|B|+1)$ possible combinations of the values of the random variables P_k, Q_k, X_k, and Y_k in order to determine the probability density of W_2, we have determined the densities of the random variables W_k for $3 \leqslant k \leqslant K$ as well. We may perform K-2 convolutions to obtain the density of $W=\sum_{k=2}^{K} W_k$. In summary, we have shown that, conditioned on knowing the signature sequence of the first receiver, it is sufficient to consider at most $(N+1)^2$ possible combinations and perform K-2 convolutions to model the other K-1 randomly generated signature sequences. It is not necessary to consider $2^{N(K-1)}$ combinations.

It is important at this point to recall again that the expression for the output statistic $Z_{out}^{(1)}$ of (4.28) is conditioned on the signature sequence of the first receiver. However, (4.28) depends on the sequence of the first receiver only through the parameter $C_{1,1}(1)$, i.e., only through the discrete aperiodic autocorrelation function of this sequence evaluated at argument 1. The probability densities of X_k and Y_k depend on the parameters $|A|$ and $|B|$, and these parameters in turn depend on the signature sequence of the first receiver only through the parameter $C_{1,1}(1)$. To see that the parameters $|A|$ and $|B|$ depend on the signature sequence of the first receiver only through the parameter $C_{1,1}(1)$, notice from (4.9), (4.22), and (4.23) that $C_{1,1}(1)$ is equal to the difference between the cardinality of the set A and the cardinality of the set B, i.e.,

$$C_{1,1}(1)=|A| - |B| \,. \tag{4.32}$$

Since $|A| + |B| = N-1$,

$$|A| = [N-1+C_{1,1}(1)]/2 \tag{4.33}$$

and

$$|B| = [N-1-C_{1,1}(1)]/2. \tag{4.34}$$

The cardinality of the set A and the cardinality of the set B can each be expressed in terms of the discrete aperiodic autocorrelation function of the signature sequence of the first receiver.

At this point another key simplification can be stated. In order to obtain (4.28), it is not necessary to condition on the random vector which models one period of the signature sequence of the first receiver. It is sufficient to condition on the single random variable $C_{1,1}(1)$, the discrete aperiodic autocorrelation function of the signature sequence of the first receiver evaluated at argument 1. This is a significant simplification because although there are 2^N possible signature sequences for the first receiver, there are only N possible values for the random variable $C_{1,1}(1)$. The 2^N signature sequences which the first receiver can use fall into N classes. The performance of the receiver is the same for all signature sequences in the same class. Combining this fact with the simplification mentioned earlier, we see that, instead of considering the 2^{NK} possible combinations of signature sequences for the K transmitters, in our simplified model we need only consider at most $N\cdot(N+1)^2$ possible combinations of values of discrete random variables and perform K-2 convolutions.

The problem of obtaining the distribution of the random variable $C_{1,1}(1)$ remains. However, we may use (4.9) to obtain

$$C_{1,1}(1) = \sum_{j=0}^{N-2} a_j^{(1)} a_{j+1}^{(1)}. \tag{4.35}$$

We may express (4.35) as

$$C_{1,1}(1) = \sum_{j=0}^{N-2} c_j^{(1)}, \tag{4.36}$$

where $c_j^{(1)} = a_j^{(1)} a_{j+1}^{(1)}$. Each c_j, for $0 \leqslant j \leqslant N-2$, is a random variable which indicates whether the next element of the first signature sequence is the same as the preceding element or different from the preceding element. If $a_j^{(1)} = a_{j+1}^{(1)}$, then $c_j^{(1)} = 1$. If $a_j^{(1)} \neq a_{j+1}^{(1)}$, then $c_j^{(1)} = -1$. Since the set of random variables $a_j^{(1)}$ for $0 \leqslant j \leqslant N-1$ is a set of independent and identically distributed random variables such that $\Pr\{a_j^{(1)} = +1\} = \Pr\{a_j^{(1)} = -1\} = 1/2$, the random variables $c_j^{(1)}$ for $0 \leqslant j \leqslant N-2$ are a set of independent and identically distributed random variables such that $\Pr\{c_j^{(1)} = +1\} = \Pr\{c_j^{(1)} = -1\} = 1/2$. Routine combinatorial arguments show that the probability density function of the discrete random variable $C_{1,1}(1)$ is given by

$$p_{C_{1,1}(1)}(j) = C\left(N-1, \frac{j+N-1}{2} \right) 2^{1-N}, \quad j = 1-N, 3-N, \ldots, N-3, N-1 \tag{4.37}$$

and

$$p_{C_{1,1}(1)}(j) = 0, \qquad \text{elsewhere.} \tag{4.38}$$

We now summarize the results we have obtained in this section. We have obtained the output statistic of the correlation receiver in the form given by equations (4.28), (4.29), (4.30), and (4.31). This form is particularly useful for performing computations. Although there are 2^{KN} possible combinations of signature sequences which could be used in the system, we have shown that to model the signature sequences it is sufficient to consider at most $N \cdot (N+1)^2$ possible combinations of values of discrete random variables and to perform K-2 convolutions. Furthermore, the random variables η, S_k, ϕ_k, $C_{1,1}(1)$, P_k, Q_k, X_k, and Y_k for k in the set $\{2, \ldots, K\}$ are a set of mutually independent random variables. This is also a significant aid in efficient computation.

4.4 Characteristic Function of the Output Statistic of the Receiver

It is sometimes preferable to work with the characteristic function of the output statistic of the receiver instead of the probability density functions of the random variables which define the output statistic. For this reason we evaluate the characteristic function of the output statistic in this section. The characteristic function of the output statistic of the receiver follows directly from the simplified expression given by (4.28), (4.29), (4.30), and (4.31). We begin by normalizing the output statistic of the receiver so that the magnitude of the desired signal component is one. We next evaluate the characteristic function of the multiple-access interference. This is then multiplied by the characteristic function of the thermal noise in order to obtain the overall characteristic function of the random component of the output statistic.

We first normalize (4.28) so that the magnitude of the desired signal component is equal to one; i.e., we divide both sides of (4.28) by $T\sqrt{P/2}$. The resulting normalized output statistic is given by

$$Z_{out}^{(N)} = \zeta + b_1^{(1)} + T^{-1}W\,, \tag{4.39}$$

where ζ is a Gaussian random variable with variance $\frac{N_0}{2TP}$ and $W = \sum_{k=2}^{K} W_k$. Since the energy per bit E_b equals PT, the variance of ζ is also given by $\frac{N_0}{2E_b}$. The characteristic function of ζ is given by

$$\Phi_\zeta(u) = \exp\left(\frac{-N_0u^2}{4E_b}\right). \tag{4.40}$$

Recall that η, and hence ζ, is a random variable that represents the effects of thermal noise at the receiver. The term $T^{-1}W$ represents the multiple access interference.

Our first step in finding the characteristic function of the multiple-access interference is to find the characteristic function of the random variable W_2, which is defined in (4.29). Suppose $S_2 = \hat{S}_2$ and $\phi_2 = \hat{\phi}_2$. Since P_2, Q_2, X_2, Y_2, S_2, and ϕ_2 are mutually independent, the characteristic function of W_2, when conditioned on the random variables S_2, ϕ_2, and $C_{1,1}(1)$, can be computed from (4.29), (4.30), and (4.31) to be

$$\Phi_{W_2}(u\,;c,\hat{S}_2,\hat{\phi}_2) = E\{e^{juW_2} \mid S_2=\hat{S}_2\,, \phi_2=\hat{\phi}_2\,, C=c\} = z(uT\,; \hat{S}_2, \hat{\phi}_2, |A|, |B|\,)\,, \tag{4.41}$$

where

$$z(u\,; s,\phi,i,j) = \cos[uT^{-1}\hat{R}_\psi(s)\cos\phi]\cos[uT^{-1}R_\psi(s)\cos\phi]\{\cos[uT^{-1}f(s)\cos\phi]\}^i\{\cos[uT^{-1}g(s)\cos\phi]\}^j, \tag{4.42}$$

and we are using a simplified notation in which $C=C_{1,1}(1)$. Averaging with respect to the random variables S_2 and ϕ_2, we find the characteristic function of W_2 when conditioned on $C_{1,1}(1)$ to be

$$\Phi_{W_2}(u;c) = \frac{2}{\pi T_c}\int_0^{T_c}\int_0^{\pi/2} \cos[u\hat{R}_\psi(s)\cos\phi]\cos[uR_\psi(s)\cos\phi]\{\cos[uf(s)\cos\phi]\}^{|A|}\{\cos[ug(s)\cos\phi]\}^{|B|}d\phi ds. \quad (4.43)$$

Notice in (4.43) that the integration is over the interval $[0,\pi/2]$ instead of the interval $[0,2\pi]$. This simplification is possible because of the structure of the integrand in (4.43) and because the cosine function is an even function.

Since conditioned on the random variable $C_{1,1}(1)$ the random variables W_k for $2 \leqslant k \leqslant K$ are mutually independent and identically distributed, the characteristic function of the random variable W conditioned on the random variable $C_{1,1}(1)$ is given by

$$\Phi_W(u;c) = \{\Phi_{W_2}(u;c)\}^{K-1} . \quad (4.44)$$

We denote the total multiple-access interference $T^{-1}W$ by Ξ. The characteristic function of the total multiple-access interference Ξ conditioned on the random variable $C_{1,1}(1)$ is given by

$$\Phi_{\Xi}(u;c) = \{\Phi_{W_2}(\frac{u}{T};c)\}^{K-1} , \quad (4.45)$$

or, using (4.43),

$$\Phi_{\Xi}(u;c) = \left\{ \frac{2}{\pi T_c}\int_0^{T_c}\int_0^{\pi/2} z(u\,;s,\phi,|A|,|B|\,)d\phi ds \right\}^{K-1} . \quad (4.46)$$

The characteristic functions which have been defined so far have been conditioned on the random variable $C_{1,1}(1)$. We can use (4.33) and (4.34) to express $|A|$ and $|B|$ in (4.46) in terms of the random variable $C_{1,1}(1)$. Next we may average with respect to the random variable $C_{1,1}(1)$ by using the probability density function of $C_{1,1}(1)$ given in (4.37) and (4.38). We find that the characteristic function of the total multiple-access interference can be expressed as

$$\Phi_{\Xi}(u) = 2^{1-N}\sum_{i=0}^{N-1} C(N-1, i)\left[\frac{2}{\pi T_c}\int_0^{T_c}\int_0^{\pi/2} z(u\,;s,\phi,i,N-1-i)\,d\phi\,ds\right]^{K-1}. \tag{4.47}$$

The characteristic function of the total multiple-access interference $\Phi_{\Xi}(u)$ has an alternative expression which is given by

$$\Phi_{\Xi}(u) = 2^{1-N}\sum_{i=0}^{N-1} C(N-1, i)[B(u,i)+B(u,i+1)]^{K-1}, \tag{4.48}$$

where

$$B(u,i) = \frac{1}{\pi T_c}\int_0^{T_c}\int_0^{\pi/2} \{\cos[uT^{-1}f(s)\cos\phi]\}^{i}\{\cos[uT^{-1}g(s)\cos\phi]\}^{N-i}d\phi ds. \tag{4.49}$$

This alternative expression can be obtained from (4.47) by applying the trigonometric identity

$$2\cos[uT^{-1}\hat{R}_{\psi}(s)\cos\phi]\cos[uT^{-1}R_{\psi}(s)\cos\phi] = \cos[uT^{-1}f(s)\cos\phi] + \cos[uT^{-1}g(s)\cos\phi] \tag{4.50}$$

to (4.47).

The characteristic function of the total random component of the normalized output statistic $Z_{out}^{(N)}$ is given by the product of the characteristic functions $\Phi_{\zeta}(u)$ and $\Phi_{\Xi}(u)$. The magnitude of the desired signal component is one.

4.5 Upper and Lower Bounds on the Average Probability of Bit Error

In this section we illustrate the use of the simplified expression for the output statistic of the receiver by obtaining upper and lower bounds on the average probability of bit error of a correlation receiver. For our illustration we choose the case of two transmitters, i.e., K=2, and a rectangular chip waveform, i.e., $\psi(t) = p_{T_c}(t)$. We choose signature sequences of length 31 (i.e., N =31). The extensions of the methods to greater numbers of interfering transmitters and longer signature sequences, as well as to other signaling techniques, are also discussed.

We proceed by first obtaining upper and lower bounds on the probability density function of the multiple-access interference Ξ conditioned on $C_{1,1}(1)$, the discrete aperiodic autocorrelation function of the signature sequence of the receiver evaluated at argument 1. We then average the upper bound with respect to the discrete random variable $C_{1,1}(1)$ in order to obtain an overall upper bound on the probability density function of the random variable Ξ. Similarly, we average the lower bound with respect to the discrete random variable $C_{1,1}(1)$ in order to obtain an overall lower bound on the probability density function of the random variable Ξ. These overall upper and lower bounds on the probability density function of the random variable Ξ are finally used to obtain the desired upper and lower bounds on the average probability of bit error.

There are key differences between the approach of this section and other previously developed approaches for obtaining the average probability of bit error of the correlation receiver. In the approach of this section, two vectors are evaluated. One vector is used to obtain an upper bound on the performance of the correlation receiver, and another vector is used to obtain a lower bound on the performance of the correlation receiver. The two bounds both become tighter and tighter as the size of the vectors which we consider grows. By using this approach, we avoid the problem of evaluating the characteristic function of the random output of the receiver and the corresponding problem of numerically evaluating integrals. Furthermore, since we obtain upper and lower bounds on the average probability of bit error, we know how accurate our evaluation of the average probability of bit error is. If the results are not accurate enough, we simply increase the size of the vectors on which our bounds are based, but we need not perform any unnecessary computations because we are doubtful about the accuracy of an approximation. We have eliminated the problem of ascertaining whether numerical integrations have been performed with enough accuracy to yield a good final result.

Another benefit of the approach which is outlined in this section is that the computation is performed in a series of steps. The total computational requirement for a specific problem is just the sum of the requirements at the several steps. The computations of several of the steps must only be performed once, but once completed, they apply to several different problems. For example, by performing a certain amount of computation, we obtain an upper and a lower bound on the probability density function of the multiple-access interference at the output of the matched filter. We then use this result to obtain upper and lower bounds on the average probability of bit error for coherent detection. We may use the same result to obtain upper and lower bounds on many different forms of detection and signaling. Much of the required computation has already been completed in order to obtain upper and lower bounds on the average probability of bit error for coherent detection. Another example of this benefit is apparent when computing the average probability of bit error for various numbers of active transmitters. Once the computation has been performed in order to determine the performance for a given number of active transmitters, some of the computation that is useful in determining the performance for other numbers of active transmitters has already been completed.

Another feature of this approach may be less obvious, but it is of great practical importance. Many of the computations which are required to obtain the upper and lower bounds on the probability density function of the multiple-access interference involve standard operations on vectors. This is also true of the remaining steps which are involved in determining upper and lower bounds on the average probability of bit error from the bounds on the probability density function of the multiple-access interference. These standard vector operations can be performed very efficiently on an array processor. Standard routines from the library of an array processor have been used in order to evaluate the performance of the system that we are using as an illustration.

4.5.1 Bounds on the Probability Density of Ξ

In this section we obtain upper and lower bounds on the probability density function of the random variable Ξ which models the multiple-access interference. Although the random variable Ξ is a continuous random variable, the upper and lower bounds which we obtain are in the form of two vectors. Such a form is possible because the probability density function of the random variable Ξ is nonzero only on an interval $[-k,k]$, where k is a constant which can be evaluated assuming the worst or the best interference conditions. Our approach is to consider $\hat{\Xi}=N\Xi$, a normalized version of Ξ, and to partition the interval $[-Nk,Nk]$ into a number N_i of subintervals. If we choose the number of subintervals per unit to be N_u, we partition the interval $[-Nk,Nk]$ into $2NN_uk$ subintervals, i.e., $N_i=2NN_uk$. We next determine two vectors of length $2NN_uk$. Each component of the first vector is an upper bound on the probability that the value of the random variable $\hat{\Xi}$ lies in a corresponding subinterval. Each component of the second vector is a lower bound on the probability that the value of the random variable $\hat{\Xi}$ lies in a corresponding subinterval. There is a one-to-one correspondence between the components of each of the two vectors and the subintervals of the partition.

To be more specific, we again consider (4.28) and (4.29) and normalize (4.28) by dividing each side of (4.28) by $T_c\sqrt{P/2}$. The normalized multiple-access interference $\hat{\Xi}$ can now be expressed as

$$\hat{\Xi} = \sum_{k=2}^{K} B_k \,, \tag{4.51}$$

where

$$B_k = T_c^{-1} W_k . \tag{4.52}$$

We consider the case of two transmitters, i.e., K=2, and the case in which the number N of chips per bit is odd. The method of obtaining bounds extends in an obvious way to the case in which the number N of chips per bit is even. Although the extension to the case of more than two active transmitters is less obvious, the extension is described in the following. For the case that we are presently considering, the normalized multiple-access interference specified in equation (4.51) reduces to the random variable B_2. From (4.52) and (4.29) and the definitions of the random variables which appear in (4.29), we see that the probability density function of the random variable B_2 is nonzero only on the interval $[-N,N]$ and symmetric about zero. We partition the interval $[-N,N]$ into $2NN_u$ subintervals and define two vectors

$\vec{u}$ and $\vec{v}$ which each consist of $2NN_u$ components. The vector $\vec{u}$ consists of the components u_i for i in the set $\{-NN_u,...,NN_u-1\}$. The component u_i is an upper bound on the probability that the value of the multiple-access interference lies somewhere in the interval $[iN_u^{-1}, (i+1)N_u^{-1}]$. Similarly, the vector $\vec{v}$ consists of the components v_i for i in the set $\{-NN_u,...,NN_u-1\}$. The component v_i is a lower bound on the probability that the value of the multiple-access interference lies somewhere in the interval $[iN_u^{-1}, (i+1)N_u^{-1}]$. Notice that because of the symmetry of the multiple-access interference $u_i=u_{-i-1}$ and $v_i=v_{-i-1}$. Our present goal is to determine the vector components u_i for i in the set $\{0,...,NN_u-1\}$ and the vector components v_i for i in the set $\{0,...,NN_u-1\}$. Because of the symmetry of the density of the multiple-access interference, we have then determined the complete vectors $\vec{u}$ and $\vec{v}$.

In the following we rely on the notion of conditional probability density functions. We express each random variable B_k, for k in the set $\{2,...,K\}$, as

$$B_k=A_k\cos\phi_k, \tag{4.53}$$

where

$$A_k=[P_k\hat{R}_\psi(S_k)+Q_kR_\psi(S_k)+X_kf(S_k)+Y_kg(S_k)]T_c^{-1}. \tag{4.54}$$

In Figure 4.2, we plot the probability density function $p_{A_2}(x)$ for the example that we are considering, i.e., for the case of a rectangular chip waveform and N=31. Since the probability density function $p_{A_2}(x)$ is symmetric about zero, we plot the density only for $x\geqslant 0$. Notice that the density consists of a continuous component that is piece-wise constant and a discrete component. In Appendix B, we show that $p_{A_2}(x\,;c)$, the probability density function of the random variable A_2 conditioned on the event that $C=C_{1,1}(1)=c$, can be expressed in terms of conditional densities that have a certain form. Each of the conditional densities is either uniform on an interval or an impulse. We can express $p_{A_2}(x\,;c)$ as

$$p_{A_2}(x\,;c)=E\{p_{A_2|\alpha}(x|\alpha\,;c)\}, \tag{4.55}$$

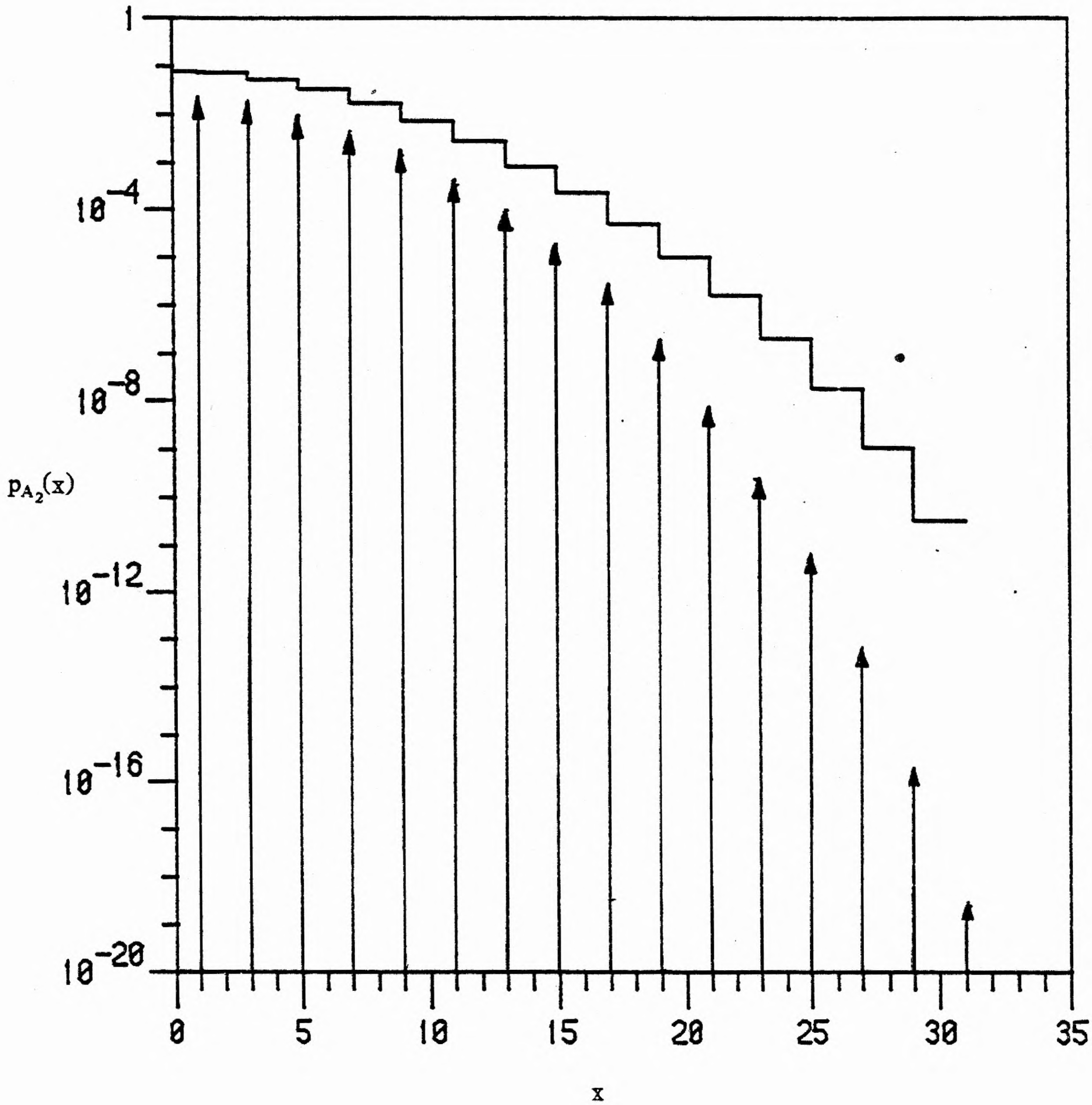

Figure 4.2. Probability density function of the random variable, A_2 (N=31, rectangular chip waveform).

where in (4.55) the expectation is with respect to the random variable α and the result is conditioned on the event that the random variable $C=C_{1,1}(1)=c$. The random variable α can be viewed as an index of several conditional probability density functions. In Appendix B, we show that the conditional probability density functions $p_{A_2|\alpha}(x|i\,;c)$ are given by

$$p_{A_2|\alpha}(x|i\,;c) = \begin{cases} \delta(x-i), & i=-N,\,-N+2,\ldots,N-2,\,N \\ \frac{1}{2}p_2(x-i+1), & i=-N+1,\,-N+3,\ldots,N-3,\,N-1\ , \end{cases} \tag{4.56}$$

where $\delta(x)$ is the unit impulse function and $p_2(x)=1$ for $0 \leqslant x \leqslant 2$, and $p_2(x)=0$, elsewhere. Notice that the conditional densities $p_{A_2|\alpha}(x|i\,;c)$ do not depend on the parameter c. The probability density of the discrete random variable α, however, does depend on the parameter c. Notice that the index α takes on values in the set $\{-N,\ldots,N\}$. We denote the discrete probability density function of the random variable α by a vector $\vec{w}(c)$ with components $w_i(c)$ for i in the set $\{-N,\ldots,N\}$. The component $w_i(c)$ is the probability that the index α is equal to i, i.e., $w_i(c)=\Pr\{\alpha=i \mid C=c\}$.

We now proceed to find the probability density function of the random variable B_2 conditioned on the random variable α. It can be shown [20] that if the random variable A_2 is uniformly distributed on an interval [a,b], where a and b are either both positive or both negative, the probability density function of the random variable B_2 is given by

$$p_{B_2}(x\,;a,b) = \begin{cases} \dfrac{1}{\pi(b-a)}\ln\left|\dfrac{b}{a}\,\dfrac{1+\sqrt{1-x^2/b^2}}{1+\sqrt{1-x^2/a^2}}\right|, & 0\leqslant|x|\leqslant a \\ \dfrac{1}{\pi(b-a)}\ln\left|\dfrac{b}{x}[1+\sqrt{1-x^2/b^2}]\right|, & a\leqslant|x|\leqslant b \\ 0, & \text{elsewhere.} \end{cases} \tag{4.57}$$

It can also be shown [20] that if A_k is equal to a nonzero constant k, the probability density function of the random variable B_2 is given by

$$p_{B_2}(x\,;k) = \begin{cases} \dfrac{1}{k\pi\sqrt{1-x^2/k^2}}, & |x|<|k| \\ 0, & \text{elsewhere.} \end{cases} \tag{4.58}$$

(Notice that in the notation which we use, the functions $p_{B_2}(x\,;a,b)$ and $p_{B_2}(x\,;k)$ are distinguished by the number of arguments that each function has.) Since the conditional densities of (4.56) are either impulses or else uniform on an interval, we may use the results of equations (4.56), (4.57), and (4.58) to obtain the conditional density of B_2 given α=i and C=c, i.e., to obtain $p_{B_2|\alpha}(x|i\,;c)$. The conditional density of B_2 given that α=i and C=c can be expressed as

$$p_{B_2|\alpha}(x|i\,;c) = \begin{cases} p_{B_2}(x\,;i)\,, & i=-N, -N+2,\ldots, N-2, N \\ p_{B_2}(x\,;i-1,i+1)\,, & i=-N+1, -N+3,\ldots, -1, 1, 3,\ldots,N-3, N-1 \\ \lim\limits_{\epsilon\to 0^+} p_{B_2}(x\,;\epsilon,1), & i=0\,. \end{cases} \tag{4.59}$$

Notice that the conditional densities $p_{B_2|\alpha}(x|i\,;c)$ do not depend on the parameter c.

We have analytically found the exact value of the conditional density of B_2 given the independent random variables α and C. We can therefore obtain the exact probability density function of the random variable B_2 given that C=c from the expression

$$p_{B_2}(x\,;c) = E_\alpha\{p_{B_2|\alpha}(x|\alpha\,;c)\}\,. \tag{4.60}$$

We may also express (4.60) in terms of the components of the vector $\vec{w}(c)$ as

$$p_{B_2}(x\,;c) = \sum_{i=-N}^{N} w_i(c)p_{B_2|\alpha}(x|i\,;c). \tag{4.61}$$

Notice that for the case of multiple interfering transmitters, the probability density function of the multiple-access interference $\hat{\Xi}$ conditioned on the parameter $C_{1,1}(1)$ can be found from (4.51) by performing K-2 convolutions since the random variables B_k for k in the set $\{2,...,K\}$ form a set of mutually independent and identically distributed random variables.

Instead of using the continuous density of (4.61), our approach is to obtain upper and lower bounds on the density of (4.61) in a discrete form before proceeding. In order to do this, we define two vectors $\vec{u}(i\,;c)$ and $\vec{v}(i\,;c)$, where i is an index in the set $\{-N,...,N\}$. The vector $\vec{u}(i\,;c)$ is the vector $\vec{u}$ that we have defined earlier given that the random variable α=i and the random variable C=c. There are $2NN_u$ components of the vector $\vec{u}(i\,;c)$. The components are denoted by $u_j(i\,;c)$ for j in the set $\{-NN_u,...,NN_u-1\}$, and the component $u_j(i\,;c)$ is an upper bound on the probability that the multiple-access interference lies somewhere in the interval $[jN_u^{-1}, (j+1)N_u^{-1}]$ given that α=i and C=c. Similarly, the vector $\vec{v}(i\,;c)$ is the vector $\vec{v}$ that we have defined earlier given that the random variable α=i and the random variable C=c. There are $2NN_u$ components of the vector $\vec{v}(i\,;c)$. The components are denoted by $v_j(i\,;c)$ for j in the set $\{-NN_u,...,NN_u-1\}$, and the component $v_j(i\,;c)$ is a lower bound on the probability that the multiple-access interference lies somewhere in the interval $[jN_u^{-1}, (j+1)N_u^{-1}]$ given that α=i and C=c.

In order to evaluate the vectors $\vec{u}(i\,;c)$ and $\vec{v}(i\,;c)$, we first consider the probability density function $p_{B_2|\alpha}(x|i\,;c)$ for i odd. In this case, $p_{B_2|\alpha}(x|i\,;c)=p_{B_2}(x\,;i)$. The function $p_{B_2}(x\,;i)$ is not only strictly increasing on the interval [0,i), but also convex on this interval. Since the function $p_{B_2}(x\,;i)$ is strictly increasing on the interval [0,i), we choose the components $v_j(i\,;c)$ to be

$$v_j(i\,;c) = \begin{cases} N_u^{-1}p_{B_2}(jN_u^{-1}\,;i), & j=0,...,iN_u-1 \\ \\ 0, & j\geq iN_u\,, \end{cases} \tag{4.62}$$

and we obtain a lower bound. Since the function is convex on the interval [0,i), we choose the components $u_j(i\,;c)$ to be

$$u_j(i\,;c) = \begin{cases} N_u^{-1}[p(jN_u^{-1}\,;i) + p(N_u^{-1}(j+1)\,;i)]/2, & j=0,\ldots,iN_u-2 \\ 1-\sum\limits_{k=0}^{iN_u-2} v_k(i\,;c), & j=iN_u-1 \\ 0, & j\geqslant N_u, \end{cases} \tag{4.63}$$

and we obtain an upper bound. Notice that since the function $p_{B_2}(x\,;i)$ is undefined at $x=i$, we must consider the definition of the vector component $u_j(iN_u-1\,;c)$ as a special case.

In Figure 4.3, we illustrate the bounds which are given by (4.62) and (4.63). We choose the case in which $N=31$, $N_u=2$, and $i=5$. In this example, the interval $[0,5]$ is divided into ten subintervals of width one half. The vector component $u_j(5;c)$ for j in the set $\{0,\ldots,9\}$ is the upper bound on the probability that the value of the random variable B_2 is somewhere in the interval $[.5j\,,.5(j+1)]$ given that $\alpha=5$ and $C=c$. It is given by the area under the top curve of Figure 4.3 that corresponds to the interval $[.5j\,,.5(j+1)]$ (except in the special case of the last component $v_9(5,c)$). The vector component $v_j(i;c)$ for j in the set $\{0,\ldots,9\}$ is the lower bound on the probability that the value of the random variable B_2 is in the interval $[.5j\,,.5(j+1)]$ given that $\alpha=5$ and $C=c$. It is given by the area under the bottom curve of Figure 4.3 that corresponds to the interval $[.5j\,,.5(j+1)]$. Notice that since the density is symmetric, we are only considering nonnegative arguments.

We next consider the probability density function $p_{B_2|\alpha}(x|i\,;c)$ for i even. In this case, $p_{B_2|\alpha}(x|i\,;c)=p_{B_2}(x\,;i-1,i+1)$ for $i\neq 0$, and $p_{B_2|\alpha}(x|i\,;c)=\lim\limits_{\epsilon\to 0^+} p_{B_2}(x\,;\epsilon,1)$ for $i=0$. The function $p_{B_2}(x\,;i-1,i+1)$ for $i\neq 0$ is strictly increasing on the interval $[0,i-1]$ and strictly decreasing on the interval $[i-1,i+1]$. Also, the function $p_{B_2}(x\,;\epsilon,1)$ for $0<\epsilon<1$ is strictly decreasing on the interval $[\epsilon,1]$. If we choose the components $v_j(i\,;c)$ for $i\neq 0$ to be

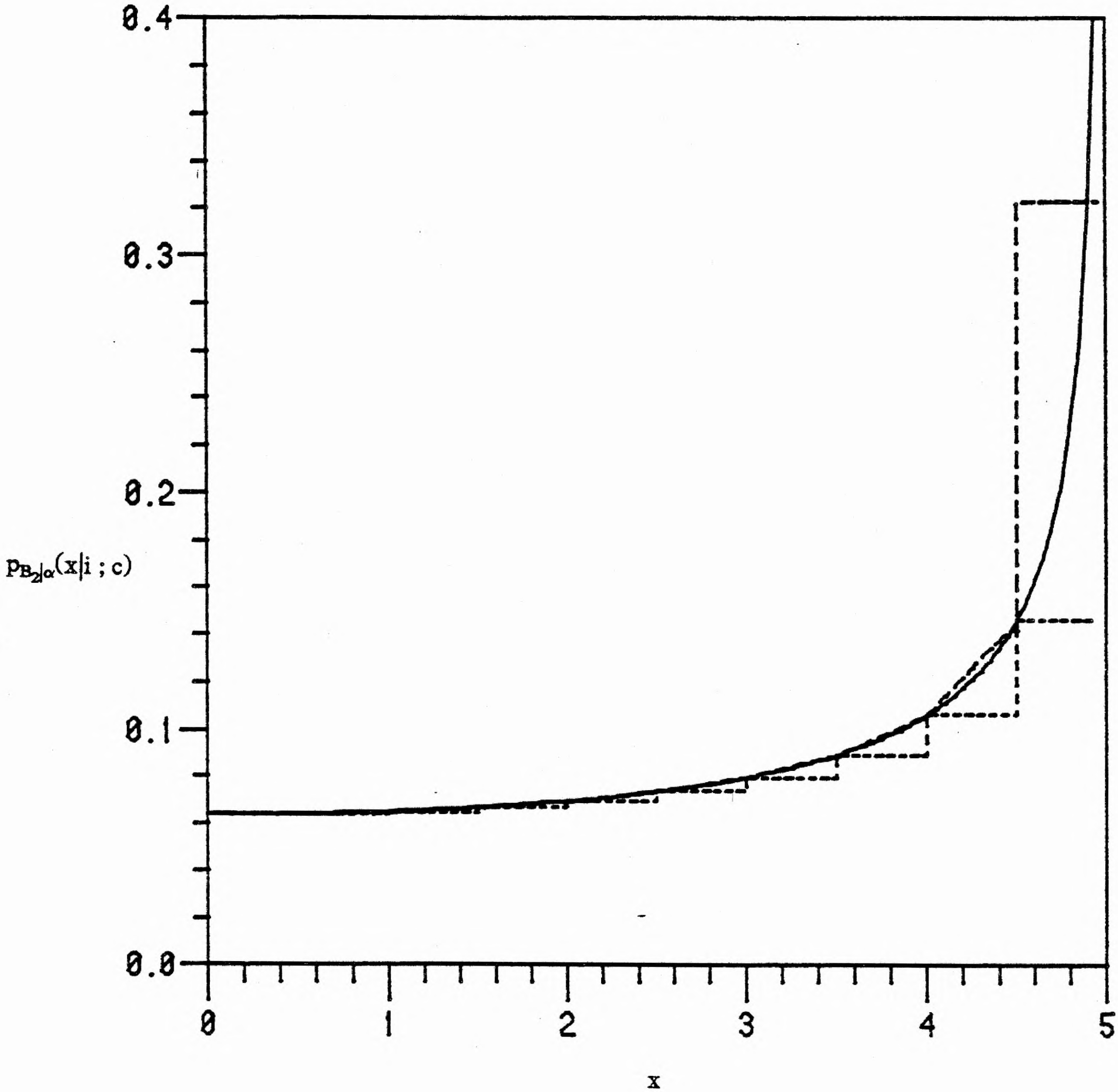

Figure 4.3. Illustration of a bounding technique on the conditional density $p_{B_2|\alpha}(x|i\ ;c)$ for $N=31$, $N_u=2$, and $i=5$.

$$v_j(i\,;c) = \begin{cases} N_u^{-1} p_{B_2}(jN_u^{-1}\,;i-1,i+1), & j=0,\ldots,(i-1)N_u-1 \\ N_u^{-1} p_{B_2}(N_u^{-1}(j+1)\,;i-1,i+1), & j=(i-1)N_u,\ldots,(i+1)N_u-1 \\ 0, & j \geqslant (i+1)N_u \end{cases} \tag{4.64}$$

and if we choose the components $v_j(0\,;c)$ to be

$$v_j(0\,;c) = \begin{cases} N_u^{-1} \lim\limits_{\epsilon\to 0^+} p_{B_2}(N_u^{-1}(j+1)\,;\epsilon,1), & j=0,\ldots,N_u-1 \\ 0, & j \geqslant N_u\,, \end{cases} \tag{4.65}$$

we obtain a lower bound. If we choose the components $u_j(i\,;c)$ for $i\neq 0$ to be

$$u_j(i\,;c) = \begin{cases} N_u^{-1} p_{B_2}(N_u^{-1}(j+1)\,;i-1,i+1), & j=0,\ldots,(i-1)N_u-1 \\ N_u^{-1} p_{B_2}(jN_u^{-1}\,;i-1,i+1), & j=(i-1)N_u,\ldots,(i+1)N_u-1 \\ 0, & j \geqslant (i+1)N_u \end{cases} \tag{4.66}$$

and if we choose the components $u_j(0\,;c)$ to be

$$u_j(0\,;c) = \begin{cases} N_u^{-1} \lim\limits_{\epsilon\to 0^+} p_{B_2}(jN_u^{-1}\,;\epsilon,1), & j=0,\ldots,N_u-1 \\ 0, & j \geqslant N_u\,, \end{cases} \tag{4.67}$$

we obtain an upper bound.

In Figure 4.4, we illustrate the bounds which are given by (4.64) and (4.66). We choose the case in which $N=31$, $N_u=2$, and $i=4$. In this example, the interval $[0,5]$ is again divided into ten subintervals of width one half. The vector component $u_j(4;c)$ for j in the set $\{0,\ldots,9\}$ is the upper bound on the probability that the value of the random variable B_2 is somewhere in the interval $[.5j\,,.5(j+1)]$

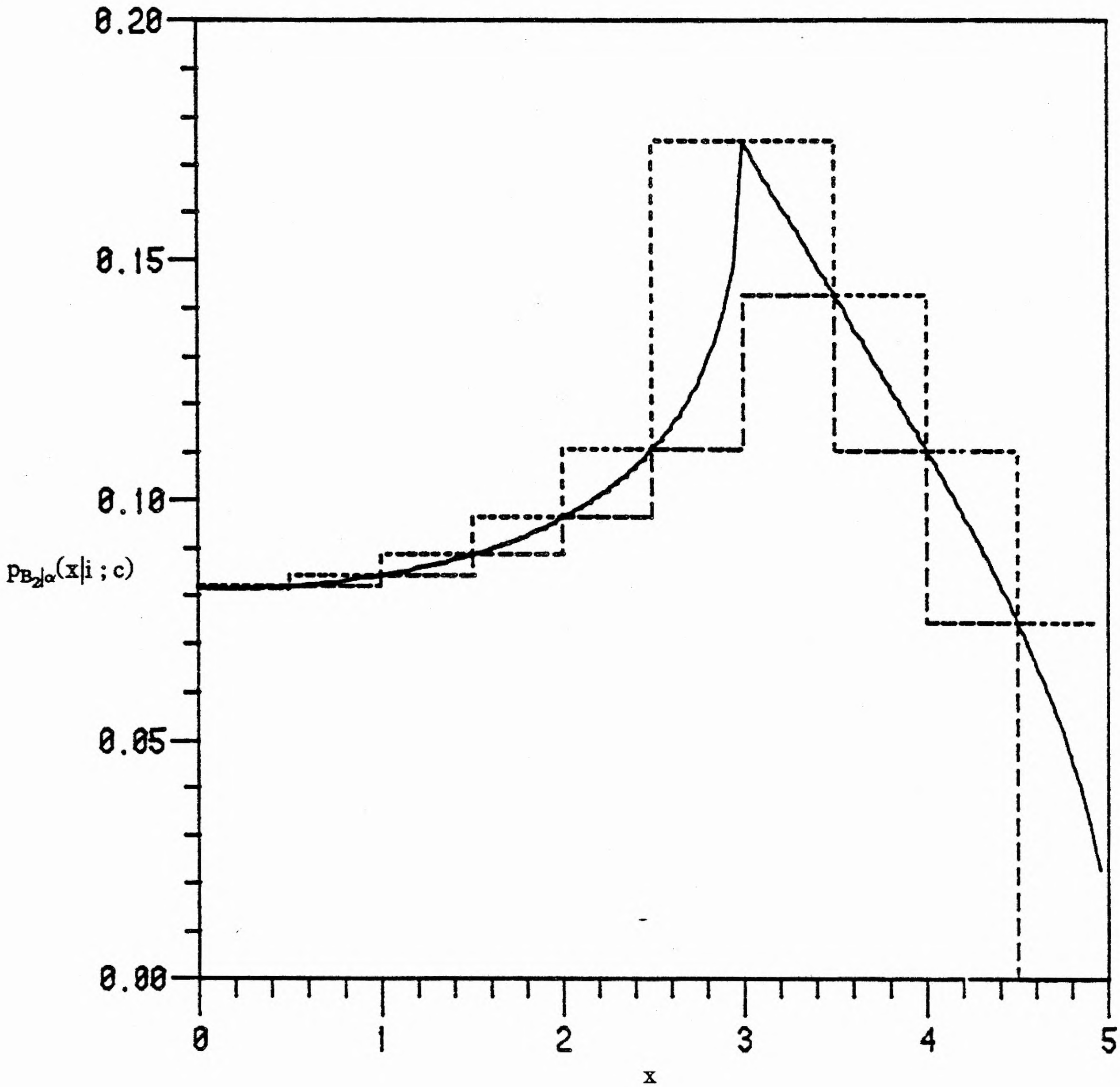

Figure 4.4. Upper and lower bounds on the conditional density $p_{B_2|\alpha}(x|i\,;c)$ for N=31, N_u=2, and i=4.

given that $\alpha=4$ and C=c. It is given by the area under the top curve of Figure 4.4 which corresponds to the interval $[.5j, .5(j+1)]$. The vector component $v_j(4;c)$ for j in the set $\{0,\ldots,9\}$ is the lower bound on the probability that the value of the random variable B_2 is in the interval $[.5j, .5(j+1)]$ given that $\alpha=4$ and C=c. It is given by the area under the bottom curve of Figure 4.4 which corresponds to the interval $[.5j, .5(j+1)]$. Notice again that since the density is symmetric we only consider nonnegative arguments.

In the preceding paragraphs we have obtained upper and lower bounds in discrete form that are conditioned on the random variables α and C. We are now in a position to obtain overall upper and lower bounds by evaluating the appropriate averages. The overall upper bound in discrete form conditioned on the event that the random variable C=c is given by

$$\vec{u}(c)=E_{\alpha}\{\vec{u}(\alpha\,;c)\}, \tag{4.68}$$

and the overall lower bound in discrete form conditioned on the event that the random variable C=c is given by

$$\vec{v}(c)=E_{\alpha}\{\vec{v}(\alpha\,;c)\}. \tag{4.69}$$

Equation (4.68) can be written in the alternative form

$$\vec{u}(c) = \sum_{i=-N}^{N} w_i(c)\vec{u}(i\,;c), \tag{4.70}$$

and (4.69) can be written in the alternative form

$$\vec{v}(c) = \sum_{i=-N}^{N} w_i(c)\vec{v}(i\,;c). \tag{4.71}$$

Since for the case of two transmitters the normalized multiple access interference $\hat{\Xi}$ conditioned on the parameter $C_{1,1}(1)$ is just the random variable B_2, we have obtained the desired upper and lower bounds on $\hat{\Xi}$ in discrete form when conditioned on the random variable $C_{1,1}(1)$. We can average these bounds with respect to the random variable $C_{1,1}(1)$ in order to obtain the overall upper and lower bounds on $\hat{\Xi}$, and hence Ξ, that we desire. The final upper bound $\vec{u}$ is given by

$$\vec{u} = E_C\{\vec{u}(C)\}\ , \tag{4.72}$$

and the final lower bound $\vec{v}$ is given by

$$\vec{v} = E_C\{\vec{v}(C)\}, \tag{4.73}$$

where E_C denotes the expectation with respect to the random variable $C=C_{1,1}(1)$.

In Figure 4.5, we plot $N_u u_i$ and $N_u v_i$ versus iN_u^{-1} (for i in the set $\{0,...,NN_u^{-1}\}$, $K=2$, $N=31$, $N_u=10$, and a rectangular chip waveform) in order to determine the shape of the probability density function of the normalized multiple-access interference $\hat{\Xi}$. For comparison, we plot on the same curve the probability density function of a Gaussian random variable with the same variance as the normalized multiple-access interference $\hat{\Xi}$. Notice that although the Gaussian density and the density of the normalized multiple-access interference $\hat{\Xi}$ agree fairly well for small arguments, the Gaussian density decays at a much faster rate than the density of $\hat{\Xi}$ for large arguments.

4.5.2 Upper and Lower Bounds on the Average Probability of Bit Error

In this section we illustrate how the vectors $\vec{u}$ and $\vec{v}$ that have been described in Section 4.5.1 can be used to obtain bounds on the performance of various receivers. As an illustration, we choose the correlation receiver.

We first obtain bounds on the performance of the correlation receiver when a constant interfering term is added to the output of the matched filter, but no multiple-access interference is present. In order to obtain these bounds, we begin with the output statistic of the correlation receiver, which is given in (4.28), and specialize to the case of one active transmitter. For convenience, we normalize the

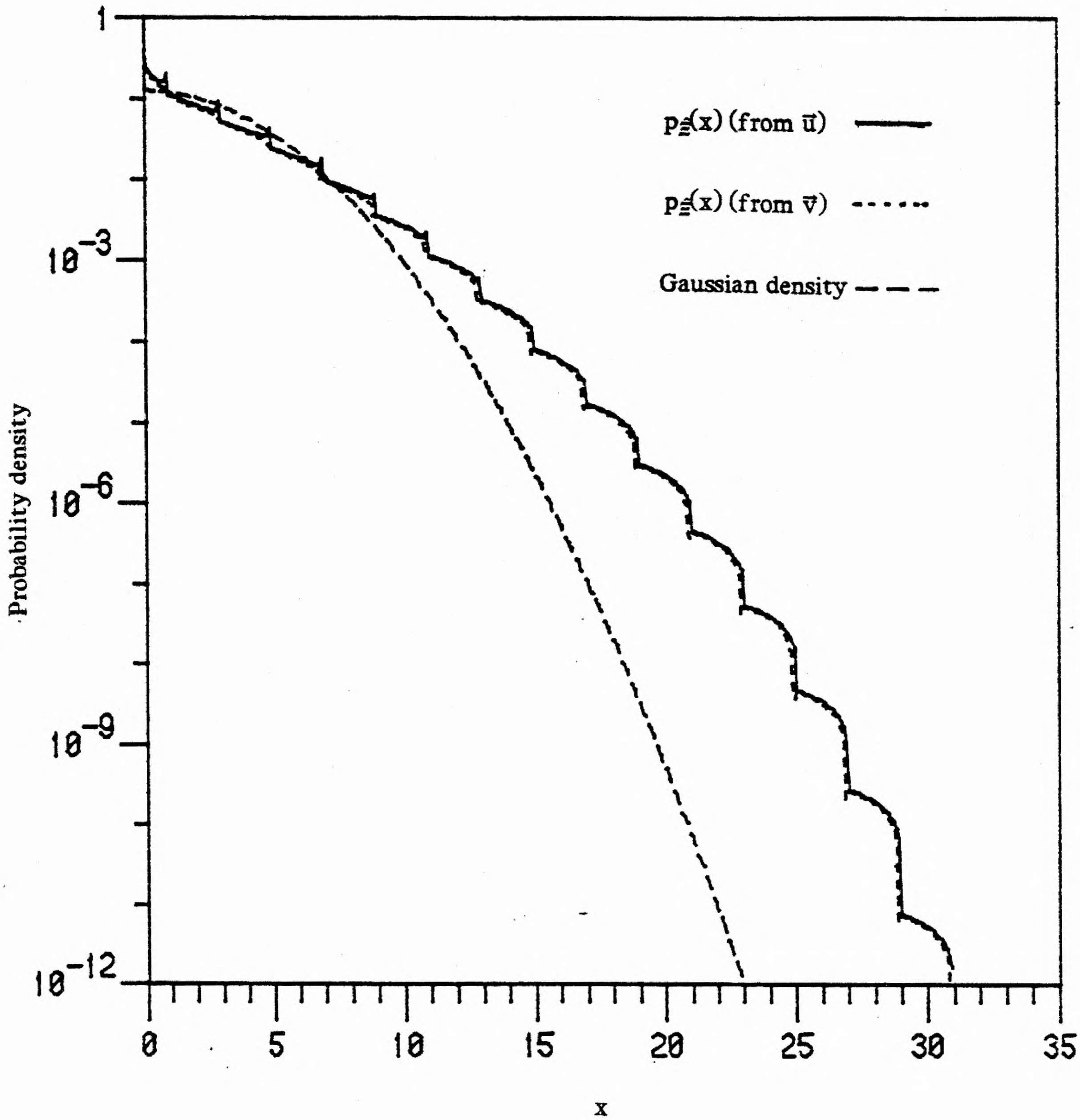

Figure 4.5. Probability density functions of $\hat{\Xi}$ corresponding to the vectors $\vec{u}$ and $\vec{v}$ compared with Gaussian density (K=2, N=31, N_u=10, rectangular chip waveform).

output statistic that is given in (4.28). We divide both sides of (4.28) by $T_c\sqrt{P/2}$ so that the desired signal component is equal to N. We see that in the case which we are considering the only random component of the output statistic is a Gaussian random variable with variance which is given by

$$\sigma^2 = \frac{N_0N^2}{2E_b}. \tag{4.74}$$

Therefore, an upper bound on the average probability of bit error is given by

$$P_E^{(U)}(k) = Q_U(\,(N+k)/\sigma\,), \tag{4.75}$$

where $Q_U(x)$ is an upper bound on the function $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-u^2/2}du$ and k is the constant interfering term. Similarly, a lower bound on the average probability of bit error is given by

$$P_E^{(L)}(k) = Q_L(\,(N+k)/\sigma\,), \tag{4.76}$$

where $Q_L(x)$ is a lower bound on the function $Q(x)$ and k is again the constant interfering term.

Although the multiple-access interference is a random quantity, we have from Section 4.5.1 an upper bound on the probability that the value of the multiple-access interference lies in an interval $[iN_u^{-1}, (i+1)N_u^{-1}]$. This probability is given by the vector component $\bar{u}_i$. To obtain bounds on the average probability of bit error of the receiver, we first assume that with probability u_i the multiple-access interference is equal to the value in the interval $[iN_u^{-1}, (i+1)N_u^{-1}]$ which yields the largest probability of bit error. Since the Q function is a strictly decreasing function, for the correlation receiver this value is iN_u^{-1}. We next use (4.75) to obtain the upper bound on the average probability of bit error that is given by

$$P_E^{(U)} = \sum_{i=-NN_u}^{NN_u-1} u_i P_E^{(U)}(iN_u^{-1}). \tag{4.77}$$

Finally, we use (4.76) to obtain the lower bound on the average probability of bit error which is given by

$$P_E^{(L)} = \sum_{i=-NN_u}^{NN_u-1} v_i P_E^{(L)}(\,(j+1)N_u^{-1}\,). \tag{4.78}$$

It is important to realize that this approach applies to other types of data modulation and detection schemes. The approach, which has been illustrated with the correlation receiver, applies when the demodulation scheme employs a filter which is matched to a signature signal and the performance of the receiver can be determined for a constant interfering term added to the output of the matched filter.

In Figure 4.6, we plot the upper and lower bounds on the average probability of bit error of the correlation receiver as a function of E_b/N_0. The results are plotted for K=2, N=31, N_u=10, and a rectangular chip waveform. For comparison, we also plot the result which we obtain if we model the normalized multiple-access interference $\hat{\Xi}$ by a Gaussian random variable with the same variance. As we expect from our observation of Figure 4.5, the Gaussian approximation is fairly good for small values of E_b/N_0, but rather bad for large values of E_b/N_0.

4.5.3 Extension to Multiple Interfering Transmitters

In Section 4.5.1, we obtained the vectors $\vec{u}$ and $\vec{v}$ that serve as bounds on the probability density function of the random variable $\hat{\Xi}$, which models the normalized multiple-access interference. However, in Section 4.5.1 we only considered the case of one interfering transmitter. In this section we describe the extension to multiple interfering transmitters.

In order to obtain the corresponding vectors $\vec{u}$ and $\vec{v}$ for multiple interfering transmitters, we begin with the vectors for a single interfering transmitter, $\vec{u}(c)$ and $\vec{v}(c)$, which are conditioned on the event that the random variable $C=C_{1,1}(1)=c$. This is necessary because without this conditioning the random variables B_k for k in the set {2,...,K}, which model the multiple-access interference from the

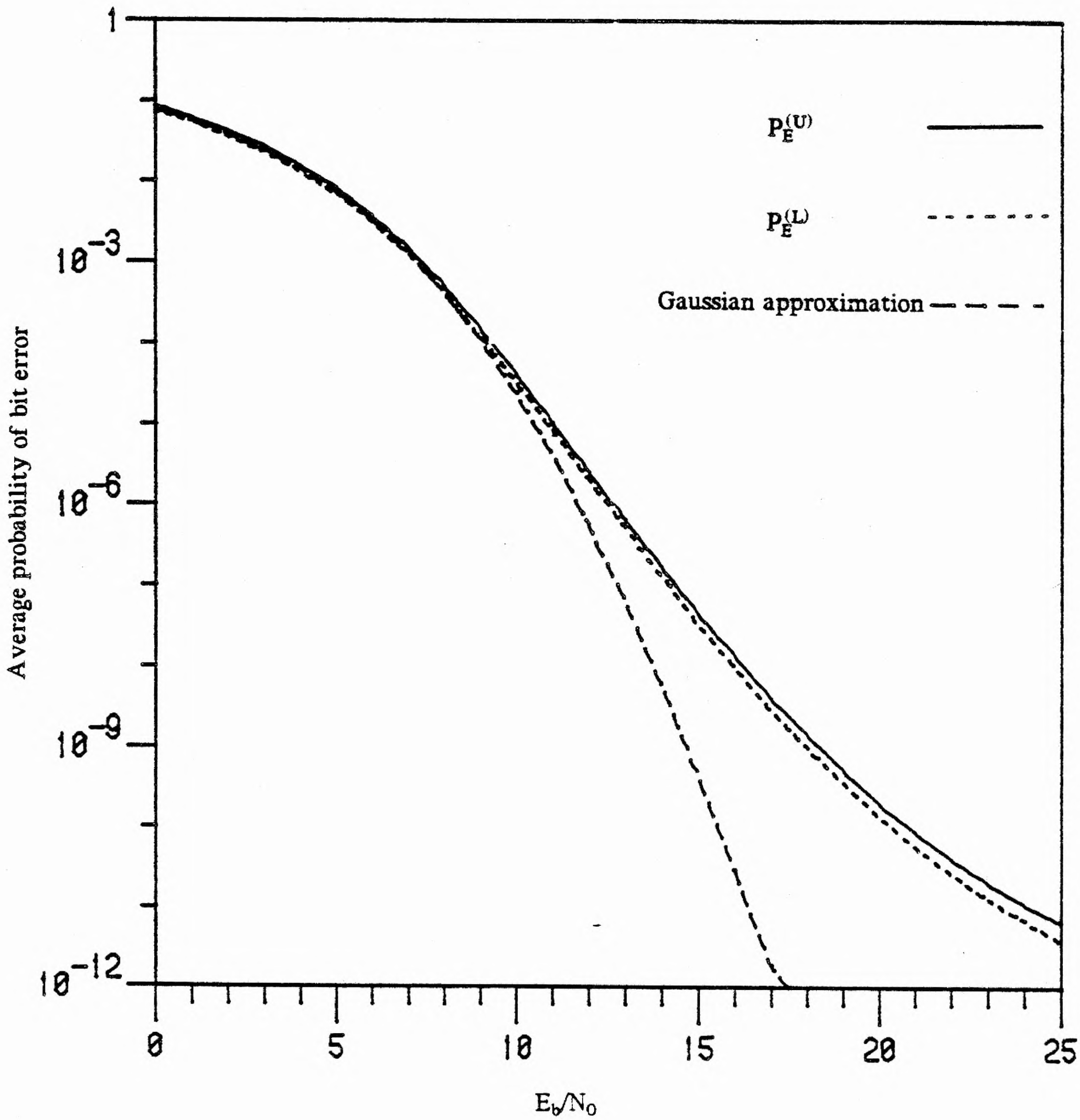

Figure 4.6. Upper and lower bounds on the average probability of bit error of the correlation receiver versus E_b/N_0 ($K=2$, $N=31$, $N_u=10$, rectangular chip waveform, random sequences).

various transmitters, are not mutually independent. Since the procedure is the same for extending the vector $\vec{v}$ to the case of multiple interfering transmitters as it is for extending the vector $\vec{u}$ to the case of multiple interfering transmitters, we describe only the extension of the vector $\vec{u}$.

We begin by recalling that the vector $\vec{u}(c)$ consists of the components $u_i(c)$ for i in the set $\{-NN_u,...,NN_u-1\}$. The component $u_i(c)$ is an upper bound on the probability that the value of the random variable B_2, modeling the interference from a single transmitter, lies in the interval $[iN_u^{-1}, (i+1)N_u^{-1}]$. We next define a set of discrete random variables D_k for k in the set $\{2,...,K\}$ such that $D_k=(i+\frac{1}{2})N_u^{-1}$ if and only if the random variable B_k lies somewhere in the interval $[iN_u^{-1}, (i+1)N_u^{-1}]$. We see that the random variable B_k never differs from the random variable D_k by more than $N_u^{-1}/2$. Finally, we define the random variables $D=\sum_{k=2}^{K} D_k$ and $B=\sum_{k=2}^{K} B_k$, and develop a relationship between the random variables D and B. To do this, we first notice that the random variable D is the sum of the K-1 terms D_k for k in the set $\{2,...,K\}$, and that the random variable B is the sum of the K-1 terms B_k for k in the set $\{2,...,K\}$. Furthermore, each term D_k can differ from the corresponding term B_k by at most $N_u^{-1}/2$. Therefore, the sum D of terms can differ from the sum B of terms by at most $(K-1)N_u^{-1}/2$.

We now proceed to obtain an upper bound on the density of the random variable D. Since the random variable B is related to the random variable D, this also serves as an upper bound on the density of the random variable B. We first note that, since the random variables B_k for k in the set $\{2,...,K\}$ form a set of mutually independent and identically distributed random variables, the random variables D_k for k in the set $\{2,...,K\}$ form a set of mutually independent and identically distributed random variables. Therefore, we may obtain the discrete density function of the random variable $D=\sum_{k=2}^{K} D_k$ by performing a series of discrete convolutions. The discrete probability density function of the random variable D is given by

$$p_D(k) = p_{D_2}(k) * p_{D_2}(k) * \cdots * p_{D_2}(k), \tag{4.79}$$

where in (4.79) there are K-2 convolutions indicated and the asterisk denotes a convolution. Since the vector $\vec{u}(c)$ is an upper bound on the discrete density function of the random variable D_2, we can replace the discrete density function $P_{D_2}(k)$ in (4.79) by the appropriate components of $\vec{u}(c)$. We then obtain an upper bound on the discrete density function of the random variable D given that the random variable C=c. We can next average with respect to the random parameter C in order to obtain an overall bound on the discrete density function of the random variable D.

Since we can obtain an upper bound on the discrete density function of D as described in the previous paragraph, we can also obtain a vector which can serve as an upper bound on the discrete density function of the random variable B. We can use this vector to obtain an upper bound on the average probability of bit error of a receiver using the approach of Section 4.5.2. The difference is that for the vector component corresponding to the subinterval $[iN_u^{-1}, (i+1)N_u^{-1}]$, we choose the value in the interval [a,b] which yields the largest probability of bit error, where $a=N_u^{-1}(2i+2-K)/2$ and $b=N_u^{-1}(2i+K)/2$.

4.6 Conclusions

We have succeeded in expressing in simplified form the output statistic of a correlation receiver used in a DS/SSMA communication system that employs randomly chosen signature sequences. Although there are 2^{KN} possible choices of signature sequences of length N for K transmitters in the system, we have obtained an expression for the output statistic of the correlation receiver that requires the consideration of no more than $N(N+1)^2$ possible combinations of the values of a set of discrete random variables and requires K-2 convolutions. We may proceed in analyzing the performance of a receiver directly from the simplified expression for the output statistic. Alternatively, we can evaluate the characteristic function of the output statistic and proceed to analyze the receiver using the characteristic function. We have shown that the characteristic function of the output statistic of the receiver can be obtained directly from the expression which we have obtained.

The simplified expression for the output statistic of the receiver has been obtained in a form that is useful for applications to packet radio systems. It is given in terms of a set of independent random variables. Included in this set are the random variables which model the relative time delays and the relative carrier phases of the interfering transmitters. If a packet consists of a series of consecutively sent data bits, the relative delays or the relative carrier phases, or both of these quantities, may be constant across the duration of a packet. Since these parameters appear explicitly in the expression that we have obtained for the output statistic of the receiver and since they are independent of the other random variables that appear in the expression, we can condition the output statistic on these random variables and perform an analysis of the system. We can then average with respect to the appropriate random variables in order to determine the overall average performance when the conditioning is not imposed.

We must consider various conditional probability density functions in the analysis of a packet radio system, and we have shown that it is useful to consider conditional probability density functions even when the added structure of the packets is not imposed on the system that we are modeling. In fact, we have seen that the components of the multiple-access interference from the various transmitters in the DS/SSMA system are not mutually independent unless the output statistic of the correlation receiver is conditioned on the random parameter $C_{1,1}(1)$. We choose to condition the output statistic of the receiver on this random parameter in order to obtain the independence of the various interfering components and to thereby facilitate the analysis of the receiver.

We have also described and given an illustration of an approach for obtaining upper and lower bounds on the average probability of bit error of a receiver used in a DS/SSMA system. This approach involves conditioning the output statistic on a set of discrete random variables and then analytically evaluating a set of conditional probability density functions. Arbitrarily tight upper and lower bounds on these probability density functions are then obtained by using properties of the density functions. The final probability density function of the multiple-access interference is determined by combining the various conditional probability density functions with the appropriate weighting. The bounds on the densities take the form of two vectors. One vector has components that are upper bounds on the probability that the value of the multiple-access interference lies in various subintervals. The other vector has components which are lower bounds on the probability that the value of the multiple-access interference lies in various subintervals. These vectors are finally used together with the results on the performance of the receiver for a constant interfering term and a single active transmitter to determine upper and lower bounds on the average probability of bit error.

The approach that we have outlined has several unique features and advantages over other methods of evaluating the average probability of bit error of the correlation receiver. First of all, instead of immediately focusing on a single scalar, such as the average probability of bit error, in this approach two vectors are first obtained which serve as upper and lower bounds on the probability density function of the multiple-access interference. Once obtained, these vectors can be used to examine the performance of other signaling and demodulation techniques which employ a filter that is matched to a signature signal and for which the performance is known given that a constant interfering term is added at the output of the matched filter. This approach allows us to determine the probability density function of the multiple-access interference, and this information is much more general and useful than a single evaluation of the probability of bit error applied to one specialized system.

A practical consideration of great importance is that the computations needed for the approach that we have outlined largely involve operations on vectors. These computations can be accomplished in an efficient manner by an array processor. Also, the computations can be separated into a series of steps so that the total required computation is just the sum of the requirements at the several steps.

One final benefit of this approach is that upper and lower bounds on both the density of the multiple-access interference and on the average probability of bit error of the receiver are determined. Furthermore, these bounds can be made arbitrarily tight. This allows a careful study of the nature of possible approximations and of the range of system parameters for which the approximations are valid. This has been illustrated with a study of the Gaussian approximation. We have seen from Figure 4.5

and Figure 4.6 that the Gaussian approximation is good for some values of E_b/N_0 and bad for other values.

CHAPTER 5

SUMMARY AND CONCLUSIONS

In this study we have examined the application of direct-sequence spread-spectrum communications to packet radio systems. In Chapter 2, we have described how part of the problem of analyzing the performance of a packet radio system is the problem of analyzing a multiple-access communication system. We have also described the specular multipath channel, which is encountered in implementations of a packet radio system, and defined a simplified model of the specular multipath channel.

In Chapter 3, we have examined the performance of a multipath-combining receiver. This receiver combines the information which is inherent in the multiple received signal replicas from an active transmitter in order to achieve performance improvements. We have shown that, if a receiver is able to perfectly determine the specular multipath channel parameters, the performance is comparable to the performance of a correlation receiver even if the transmitted power is greatly reduced. The gains which are possible are large enough that we expect improvements in practical systems that are not able to determine the specular multipath channel parameters with perfect accuracy.

We have seen that the multiple-access capability of the system which consists of a multipath-combining receiver and a specular multipath channel is about the same as we would expect from a system consisting of a simple correlation receiver and an AWGN channel. Although the multiple signal paths from an interfering transmitter to a receiver increase the multiple-access interference, the added interference is compensated by the increased received signal power that results from the multiple signal paths from the desired transmitter. If the simple correlation receiver is used in the system, however, or if the receiver is unable to ascertain the channel parameters, the multiple-access capability of a system with a specular multipath channel is greatly degraded.

We have also identified key parameters of the signature subsequences which influence the multiple-access capability of the system and the immunity of the system to intersymbol interference.

In fact, a comparison of the corresponding results for the AWGN channel and our results shows that the key parameters of the signature subsequences that influence the multiple-access capability of the system are similar to the key parameters that have been identified for the AWGN channel.

We have described two approximations to the average probability of bit error of the multipath-combining receiver. The first approximation requires very little computation. The second approximation, although requiring greater amounts of computation, is a better approximation for some system conditions.

In Chapter 4, we have simplified the model of the system to consider the multiple-access channel. We have succeeded in expressing in simplified form the output statistic of a correlation receiver which is used in a DS/SSMA communication system that employs randomly chosen signature sequences. We have shown that the characteristic function of the output statistic of the receiver can be obtained directly from the expression that we have obtained.

The simplified expression for the output statistic of the receiver has been obtained in a form that is useful for applications to packet radio systems. It is given in terms of a set of independent random variables. Included in this set are the random variables which model the relative time delays and the relative carrier phases of the interfering transmitters. If a packet consists of a series of consecutively sent data bits, the relative delays or the relative carrier phases, or both of these quantities, may be constant across the duration of a packet. Since these parameters appear explicitly in the expression that we have obtained for the output statistic of the receiver and since they are independent of the other random variables that appear in the expression, we can condition the output statistic on these random variables and perform an analysis of the system. We can then average with respect to the appropriate random variables in order to determine the overall average performance when the conditioning is not imposed.

We have shown that it is useful to consider conditional probability density functions even when the added structure of the packets is not imposed on the system that we are modeling. In fact, we have seen that the components of the multiple-access interference from the various transmitters in the

DS/SSMA system are not mutually independent unless the output statistic of the correlation receiver is conditioned on the random parameter $C_{1,1}(1)$. We have chosen to condition the output statistic of the receiver on this random parameter in order to obtain the independence of the various interfering components and to thereby facilitate the analysis of the receiver.

We have also described and given an illustration of an approach for obtaining upper and lower bounds on the average probability of bit error of a receiver used in a DS/SSMA system. Bounds on the probability density function of the multiple-access interference are obtained in the form of two vectors. One vector has components that are upper bounds on the probability that the value of the multiple-access interference lies in various subintervals. The other vector has components which are lower bounds on the probability that the value of the multiple-access interference lies in various subintervals. These vectors are used together with the results on the performance of the receiver for a constant interfering term and a single active transmitter to determine upper and lower bounds on the average probability of bit error.

The approach that we have outlined has several unique features and advantages over other methods of evaluating the average probability of bit error of the correlation receiver. First of all, instead of immediately focusing on a single scalar, such as the average probability of bit error, in this approach two vectors are first obtained which serve as upper and lower bounds on the probability density function of the multiple-access interference. Once obtained, these vectors can be used to examine the performance of other signaling and demodulation techniques that employ a filter which is matched to a signature signal and for which the performance is known given that a constant interfering term is added at the output of the matched filter. This approach allows us to determine the probability density function of the multiple-access interference, and this information is much more general and useful than a single evaluation of the probability of bit error applied to one specialized system.

A practical consideration of great importance is that the computations needed for the approach that we have outlined largely involve operations on vectors. These computations can be accomplished in an efficient manner by an array processor. Also, the computations can be separated into a series of steps so that the total required computation is just the sum of the requirements at the several steps.

One final benefit of this approach is that upper and lower bounds on both the density of the multiple-access interference and on the average probability of bit error of the receiver are determined. Furthermore, these bounds can be made arbitrarily tight. This allows a careful study of the nature of possible approximations and of the range of system parameters for which the approximations are valid. This has been illustrated with a study of the Gaussian approximation. We have seen that the Gaussian approximation is good for some values of E_b/N_0 and bad for other values. This study has explored the trade-off between the amount of computation required for various evaluations of the average probability of bit error and the accuracy of the evaluations.

APPENDIX A

OUTPUT STATISTIC FOR QUATERNARY SYSTEMS

In this appendix we state the expression for the output statistic of a correlation receiver that is used in the quadrature channel of a DS/SSMA system that employs quaternary modulation. The expression for the in-phase channel is similar. The system model is the model that has been described in [3] for the case of randomly generated signature sequences. This model includes QPSK, SQPSK, and MSK communication systems.

The analysis of such a system parallels the analysis that yields (4.28) and (4.47). The output statistic of the correlation receiver of the quadrature channel that corresponds to (4.28) is given by

$$Z_{out}^{(1)} = \eta + b_1^{(1)} T\sqrt{P/2} + \sqrt{P/2}\sum_{k=2}^{K} W_k, \tag{A.1}$$

where

$$\begin{aligned} W_k = {} & [P_{2k-1}\hat{R}_\psi(S_k)+Q_{2k-1}R_\psi(S_k)+X_{2k-1}f(S_k)+Y_{2k-1}g(S_k)]\cos\phi_k \\ & + [P_{2k}\hat{R}_\psi(\tilde{S}_k)+Q_{2k}R_\psi(\tilde{S}_k)+X_{2k}f(\tilde{S}_k)+Y_{2k}g(\tilde{S}_k)]\sin\phi_k \end{aligned} \tag{A.2}$$

and in (A.2) the argument $\tilde{S}_k = [S_k+t_0]$ modulo T_c. Recall that we are using t_0 to represent the time offset of the two quadrature channels. Typically, we assume $t_0 = T_c/2$. The random variables X_{2k-1} and X_{2k} for k in the set $\{2,\ldots,K\}$ both have a discrete probability density function which is given by (4.30). The random variables Y_{2k-1} and Y_{2k} for k in the set $\{2,\ldots,K\}$ both have a discrete probability density function which is given by (4.31). The random variables that we have not explicitly defined in this paragraph have the same definitions as in Section 4.3. For each k, the random variables η, S_k, ϕ_k, $C_{1,1}(1)$, P_{2k-1}, P_{2k}, Q_{2k-1}, Q_{2k}, X_{2k-1}, X_{2k}, Y_{2k-1}, and Y_{2k} are a set of mutually independent random variables. Again, by conditioning on the parameter $C_{1,1}(1)$, we obtain the independence of the random variables W_k for k in the set $\{2,\ldots,K\}$.

Similar to (4.46), the characteristic function of the multiple-access interference when conditioned on the event that C=c is given by

$$\Phi_{\Xi}(u;c) = \left\{ \frac{2}{\pi T_c} \int_0^{T_c} \int_0^{\pi/2} z(u\,;\,s,\,\phi,\,|A|,\,|B|\,)d\phi ds \right\}^{K-1}, \tag{A.3}$$

where now the function $z(u\,;s,\phi,i,j)$ is given by

$$z(u\,;s,\phi,i,j) = \cos[uT^{-1}\hat{R}_{\psi}(s)\cos\phi]\cos[uT^{-1}R_{\psi}(s)\cos\phi]\{\cos[uT^{-1}f(s)\cos\phi]\}^{i}\{\cos[uT^{-1}g(s)\cos\phi]\}^{j}$$

$$\cdot\cos[uT^{-1}\hat{R}_{\psi}(\tilde{s})\sin\phi]\cos[uT^{-1}R_{\psi}(\tilde{s})\sin\phi]\{\cos[uT^{-1}f(\tilde{s})\sin\phi]\}^{i}\{\cos[uT^{-1}g(\tilde{s})\sin\phi]\}^{j}, \tag{A.4}$$

where $\tilde{s}=[s+t_0]$ modulo T_c. Similar to (4.47), the overall characteristic function of the multiple-access interference is given by

$$\Phi_{\Xi}(u) = 2^{1-N}\sum_{i=0}^{N-1} C\,(N-1,i) \left\{ \frac{2}{\pi T_c} \int_0^{T_c} \int_0^{\pi/2} z(u\,;\,s,\,\phi,\,i,\,N-1-i\,)d\phi ds \right\}^{K-1}. \tag{A.5}$$

APPENDIX B

FORM OF CONDITIONAL DENSITY OF A_2

In (4.54), we defined the random variable

$$A_2=[P_2\hat{R}_\psi(S_2)+Q_2R_\psi(S_2)+X_2f(S_2)+Y_2g(S_2)]T_c^{-1}. \tag{B.1}$$

In this appendix, we show that the probability density function of this random variable can be expressed in terms of conditional probability density functions $p_{A_2|\alpha}(x|i\,;c)$ that have a certain form. The expression is given by

$$p_{A_2}(x\,;c)=E\{p_{A_2|\alpha}(x|\alpha\,;c)\}, \tag{B.2}$$

where the expectation is with respect to the random variable α. The conditional probability density functions $p_{A_2|\alpha}(x|i\,;c)$ of (B.2) for the case in which N is odd are given by

$$p_{A_2|\alpha}(x|i\,;c)=\begin{cases}\delta(x-i), & i=-N,\,-N+2,\ldots,N-2,\,N\\ \frac{1}{2}p_2(x-i+1), & i=-N+1,\,-N+3,\ldots,N-3,\,N-1\,,\end{cases} \tag{B.3}$$

where $\delta(x)$ is the unit impulse function and $p_2(x)=1$ for $0\leqslant x\leqslant 2$, and $p_2(x)=0$, elsewhere.

In order to show this, we first establish that $|A_2|\leqslant N$. First notice that $|T_c^{-1}f(S_2)|\leqslant 1$ and $|T_c^{-1}g(S_2)|\leqslant 1$ and recall from (4.30) and (4.31) that $|X_2|\leqslant|A|$ and $|Y_2|\leqslant|B|=N-1-|A|$. This means that $|[X_2f(S_2)+Y_2g(S_2)]T_c^{-1}|\leqslant N-1$. Also, since the term $[P_2\hat{R}_\psi(S_2)+Q_2R_\psi(S_2)]$ is equal to $f(S_2)$, $-f(S_2)$, $g(S_2)$, or $-g(S_2)$, $|[P_2\hat{R}_\psi(S_2)+Q_2R_\psi(S_2)]T_c^{-1}|\leqslant 1$. Hence, $|A_2|\leqslant N$.

Next, we notice that for any possible values of the discrete random variables P_2, Q_2, X_2, and Y_2, the expression for the random variable A_2 has the form

$$A_2=[af(S_2)+bg(S_2)]T_c^{-1}, \tag{B.4}$$

where a and b are integers. For the rectangular chip waveform, $f(s)=T_c$ for s in the interval $[0,T_c]$ so the expression of (B.4) reduces to

$$A_2=a+bg(S_2)T_c^{-1}. \tag{B.5}$$

Furthermore, since S_2 is uniformly distributed on the interval $[0,T_c]$, $g(S_2)T_c^{-1}$ is uniformly distributed on the interval [-1,1] for the rectangular chip waveform. Hence, if b=0, the random variable A_2 is a constant. If $b\neq0$, the random variable A_2 is uniform on an interval. By utilizing the densities of (4.30) and (4.31), it can be shown that the transition points from one constant level to another occur at the odd integers when N is odd. We can therefore express the probability density function of A_2 as in (B.2) and (B.3).

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